Mastery Development

September 1990

Catalog

and

Price List

MASTERY DEVELOPMENT

P.O. BOX 3126 REDMOND, WA 98073
(206) 885-6927 FAX (206) 885-7189

Math Facts Tracker™

The Math Facts Tracker program uses a game approach to drill students and track their progress on each of the 100 math facts from the four basic operations: addition, subtraction, multiplication, and division. Extensive tracking features are included to ensure that each student is working at the appropriate speed and level of difficulty. Reporting features print individual progress reports for students and parents and class summary reports for teachers. The Math Facts Tracker program was designed on the basis of comprehensive research involving the ability of children to learn math facts. The user's manual (accompanies purchased disks) contains a literature review that demonstrate how children move from productions to strategies, and then to recall or automated procedures when learning math facts. Additionally, a booklet entitled *Helping Children with Math Facts*, provide parents and teachers with game and activity ideas that assist students in learning math facts.

Don't hand the Math Facts Tracker disk to your students as if it were just another computer game. First, take a minute to discover the tracking and reporting features built in to this amazing program and how to set up the disk for its most effective use.

Take a minute to review the many features of the Math Facts Tracker program, and then browse through the two reports on the next page.

Tracking Features

- Tracks individual progress for up to 50 students per disk.
- Adjusts the rate of assignment and difficulty level to meet individual ability.
- Adjusts the presentation time to meet individual ability.
- Identifies and lists the facts that give students the most trouble.
- Reviews all assigned facts continuously, emphasizing the "trouble facts."
- Shows status (mastered, hesitates, or incorrect) for each fact.
- Shows student progress over the last ten sessions.

Other Features

- The three games can be assigned to any skill or turned off altogether.
- The user's manual explains theory and operation of the program's tracking capabilities and contains a full literature review of the learning of math facts.
- The parent's manual contains game and activity ideas which emphasize math fact strategies and are keyed to each student's individual progress report.
- The personalized worksheets contain only those problems which are appropriate for each student.
- The parent's report is easy to read and interpret (see "Report for Parents" on the next page).

Follow the steps below to ensure maximum benefit from the Math Facts Tracker program.

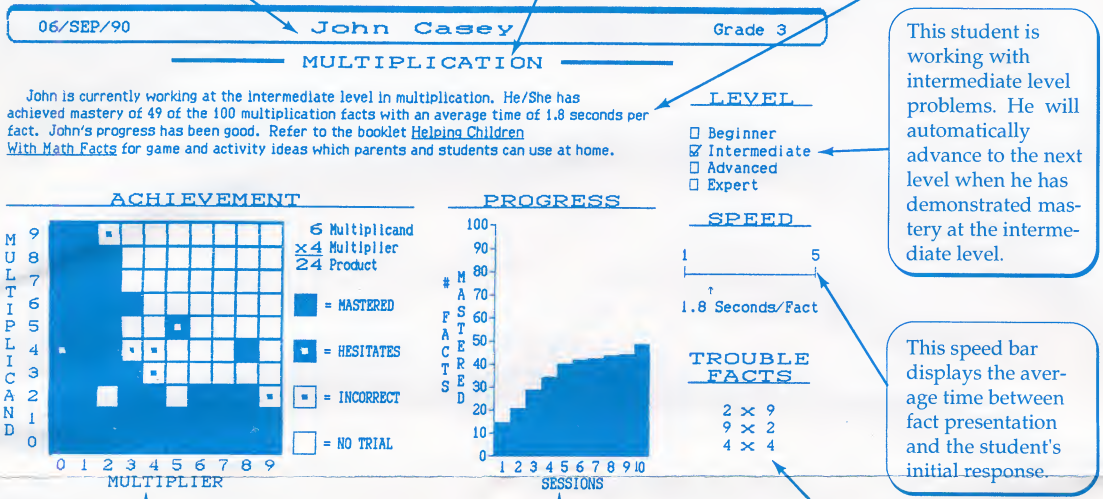
1. Boot the disk and go to the Tracking Options Menu.
2. Use option 5 to create a password. Record your new password.
3. Use option 2 to add a file for each of your students.
4. Let your students use the disk.
5. Return periodically to the Tracking Options Menu to print reports or practice sheets for your students to take home.

REPORT FOR PARENTS

Each student receives his/her own report. This report isspecifically designed for students to take home to parents.

This report shows progress on the 100 multiplication facts. Other reports can also be printed for addition, subtraction, and division.

This paragraph summarizes each section of the report. It is especially useful for those parents who do not wish to interpret charts and graphs.



This student is working with intermediate level problems. He will automatically advance to the next level when he has demonstrated mastery at the intermediate level.

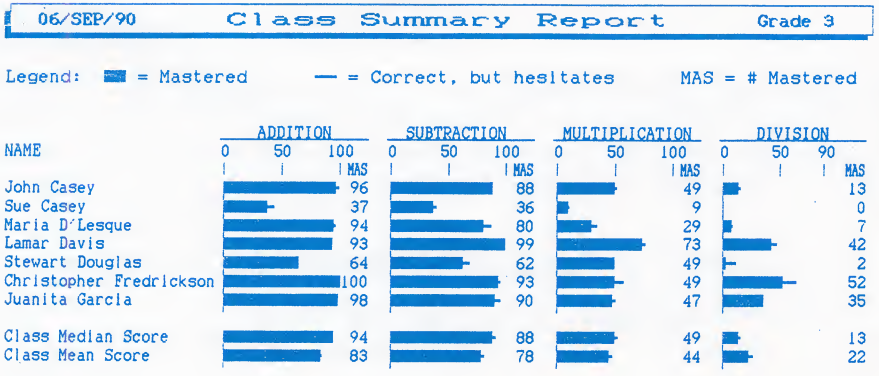
This speed bar displays the average time between fact presentation and the student's initial response.

This achievement chart shows the status (mastered, hesitates, or incorrect) of each multiplication fact.

This progress chart shows the total number of mastered facts for the last ten sessions.

This "trouble facts" list displays the facts that the student answers incorrectly on a regular basis.

REPORT FOR TEACHERS


06/SEP/90
Class Summary Report
Grade 3
Legend: = Mastered — = Correct, but hesitates MAS = # Mastered
NAME
ADDITION
SUBTRACTION
MULTIPLICATION
DIVISION
MAS
John Casey 96 88 49 13
Sue Casey 37 36 9 0
Maria D'Lesque 94 80 29 7
Lamar Davis 93 99 73 42
Stewart Douglas 64 62 49 2
Christopher Fredrickson 100 93 49 52
Juanita Garcia 98 90 47 35
Class Median Score 94 88 49 13
Class Mean Score 83 78 44 22

This report gives the teacher an "achievement snapshot" for each student. The wide bar shows the number of facts each student has mastered. The thin bar shows additional facts which the student answered correctly but computed with an inefficient method.

KEYBOARDING KLASS™

The Keyboarding Klass program is designed to teach students basic keyboarding proficiency in a minimum amount of time. This program consists of an introduction to correct typing form, six lesson-drill-game sequences, and a final drill. The major part of this program is the six lesson-drill-game sequences. Students learn new finger positions in the lessons and then practice these skills in the accompanying drills and games. Students must perform well in the drill portion to earn the game tokens which are required to play the keyboarding games. The teacher's disk utilizes a copy of the final drill to track the students' scores and progress. The teacher's disk also prints reports to show individual and class progress. This disk stores data for 500 students and 50 classes. Only one teacher's disk is needed per computer lab.

HARDWARE REQUIREMENTS.....	Apple IIe, IIc, or IIgs with 64K memory
DEMO VERSION INCLUDES.....	Program disk (demo version)
PERMANENT VERSION INCLUDES.....	Program disk, user's manual, Typing Tips poster Catalog item #9
TEACHER'S VERSION INCLUDES.....	Teacher's management disk Catalog item #10

MASTERY ARITHMETIC GAMES™

Mastery Arithmetic Games uses a game approach to teach students the following math concepts: numeration, number relations, addition, subtraction, multiplication, division, rounding and estimation, story problems, number theory, and simple fractions. This program is easy to use--just hand it over to your students and let them boot it up. A menu instantly appears prompting them to select a game. Because all of the games are self-explanatory, students learn them quickly and easily by playing them. The permanent copy allows the teacher to toggle the sound and view and reset the Hall of Fame.

HARDWARE REQUIREMENTS.....	Apple II+, IIe, IIc, or IIgs with 64K memory
DEMO VERSION INCLUDES.....	Program disk (demo version Games 1-6))
PERMANENT VERSIONS INCLUDES...	Program disk games 1-5 Catalog item #1
	Program disk games 1-10 Catalog item #3

Geo-Race U. S. A.™

Geo-Race U.S.A. uses a game approach to motivate students to learn the geography of the United States. It teaches states, capitals, cities, and major points of interest. It combines knowledge, skill, strategy, and luck as students race around the United States trying to land on all of their destinations before their opponents. It has three levels of expertise: beginner, intermediate, and advanced.

Write us a letter to let us know which Geo-Race product you would like us to produce next. Europe? Asia? South America?

HARDWARE REQUIREMENTS..... Apple IIe, IIc, or IIgs with 128K memory
DEMO VERSION INCLUDES..... Program disk (demo version)
PERMANENT VERSION INCLUDES..... Program disk Catalog item #12

The Writing Room™

The Writing Room is an easy-to-use, yet very comprehensive, writing program. It includes a word processor, spell checker, grammar checker, and pre-writing routine which helps students organize their thoughts before they start writing. Students who learn to use this writing package will be able to easily check their papers for errors and re-edit the sections they wish to change. Those who purchase a permanent copy of this program will also receive complete lesson materials including assignments, step-by-step instructions, quizzes, and overhead transparency masters to assist in training new students on how to use this product.

The demonstration version of this program is available free upon request.

HARDWARE REQUIREMENTS..... Apple IIe, IIc, or IIgs with 128K memory
DEMO VERSION INCLUDES..... Program disk (demo version), File/Dic disk, small user's manual
PERMANENT VERSION INCLUDES... Program disk, File/Dictionary disk, large user's manual, teacher's manual, overhead transparency masters Catalog item #5

Mastery Development

FLOPPY DISK PRICE LIST

1.	MASTERY ARITHMETIC GAMES 1-5	1 DISK	$ 39.95
2.	MASTERY ARITHMETIC GAMES 1-5 LAB PACK	6 DISKS	$ 114.00
3.	MASTERY ARITHMETIC GAMES 1-10	1 DISK	$ 69.95
4.	MASTERY ARITHMETIC GAMES 1-10 LAB PACK	6 DISKS	$ 192.00
5.	THE WRITING ROOM	1 DISK	$ 69.95
6.	THE WRITING ROOM LAB PACK	6 DISKS	$ 192.00
7.	THE WRITING ROOM FILE & DICTIONARY DISKS	25 DISKS	$ 50.00
8.	THE WRITING ROOM FILE & DICTIONARY DISKS	50 DISKS	$ 87.50
9.	KEYBOARDING KLASS PROGRAM DISK	1 DISK	$ 49.95
10.	KEYBOARDING KLASS TEACHER'S DISK	1 DISK	$ 19.95
11.	KEYBOARDING KLASS LAB PACK	6 DISKS & 1 TEACHER DISK	$ 168.00
12.	GEO-RACE USA	1 DISK	$ 49.95
13.	GEO-RACE USA LAB PACK	6 DISKS	$ 168.00
14.	MATH FACTS TRACKER	1 DISK	$ 49.95
15.	MATH FACTS TRACKER 6-PACK	6 DISKS	$ 168.00

Reduced pricing and site licenses are available for schools wishing to purchase complete lab sets. You will need to call us since these prices are based on the total number of disks required.

All programs are distributed on 5 1/4-inch disks. Please see the preceding pages for the specific hardware requirements for each program. Orders may be placed with a purchase order, school or personal check, or a Visa/MasterCard number. We will gladly accept phone and fax orders. All orders are shipped the same day they are received. Please allow one week for delivery.

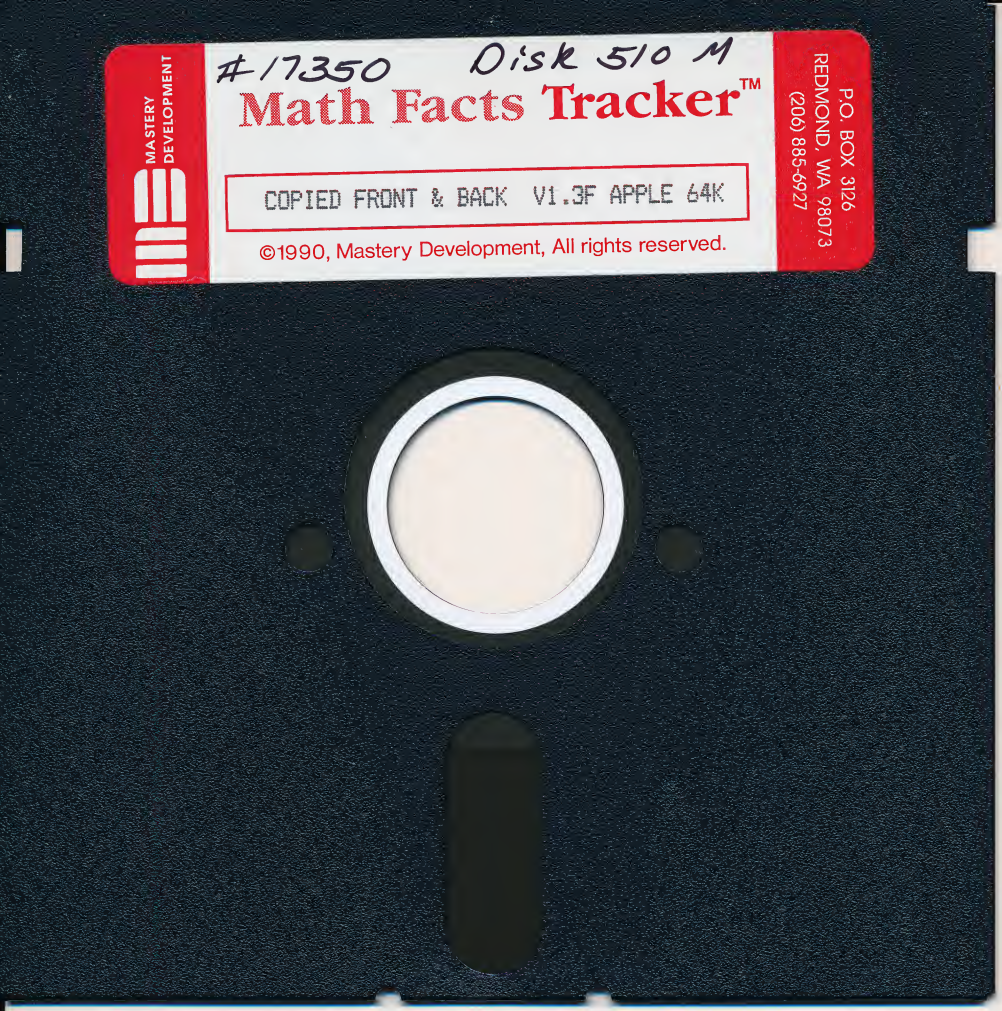
INTERESTED IN NETWORKING YOUR COMPUTER LAB?

Mastery Development has created a network system designed especially for your school's computer lab. It is fast, inexpensive, easy to use, and comes loaded with great software. Call or write for more information.

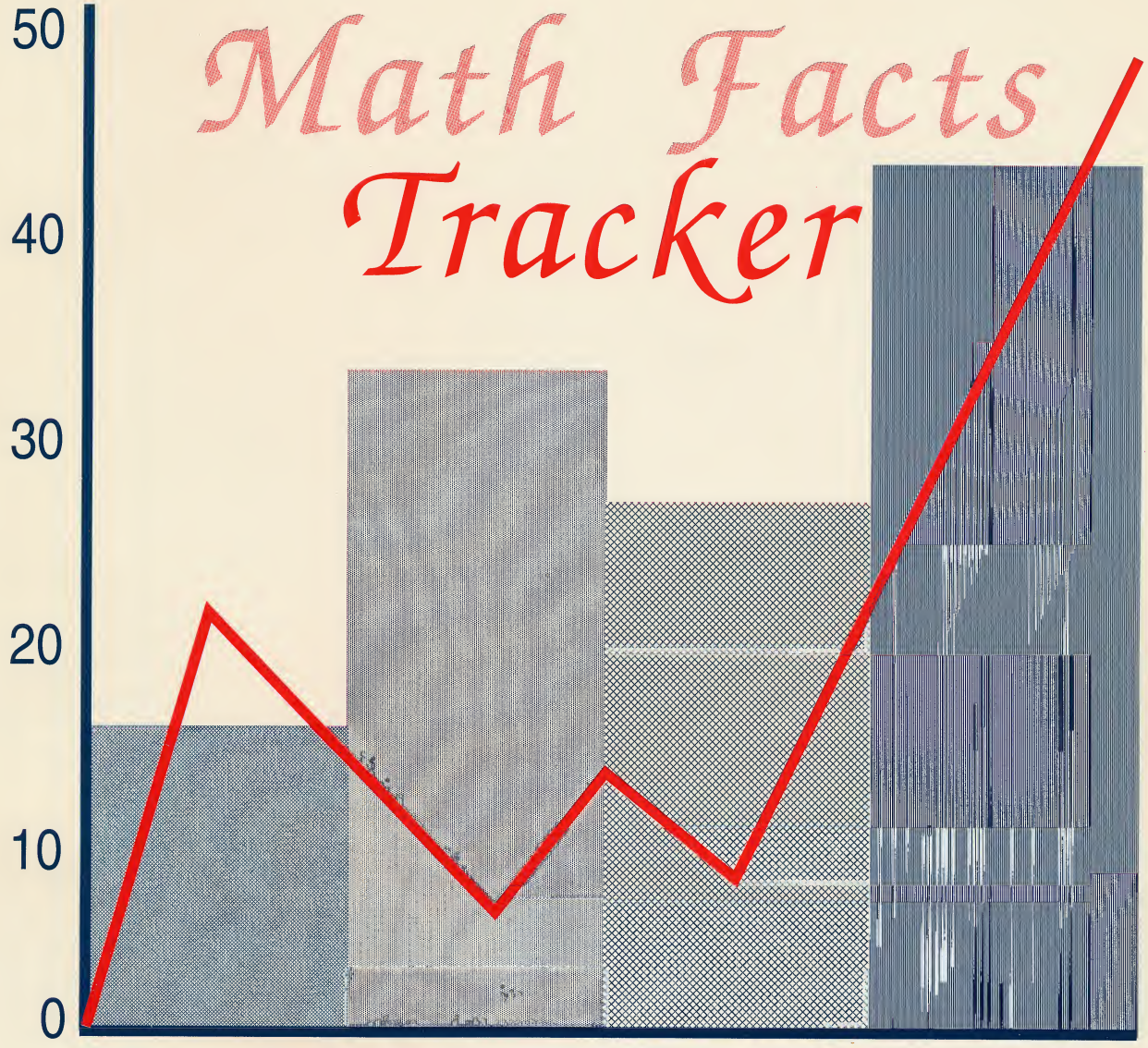
MASTERY DEVELOPMENT

P.O. BOX 3126 REDMOND, WA 98073

(206) 885-6927 FAX (206) 885-7189


#17350
Disk 510 M
Math Facts Tracker™
COPIED FRONT & BACK V1.3F APPLE 64K
©1990, Mastery Development, All rights reserved.
MASTERY DEVELOPMENT
P.O. BOX 3126
REDMOND, WA 98073
(206) 885-6927

Disk
510
M



Math Facts Tracker

Teacher's Guide & User's Manual

NORTH PARK MEDIA CENTER
COLUMBIA HEIGHTS SCHOOLS

Mastery Development

Math Facts Tracker™

Teacher's Guide & User's Manual

NORTH PARK MEDIA CENTER
COLUMBIA HEIGHTS SCHOOLS

Revision 1.0
April, 1991

Mastery Development • P. O. Box 3126 • Redmond, WA • 98073 • (206) 885-6927

Copyright Notice: The *Math Facts Tracker* software, reports, and Teacher's Guide & User's Manual are copyrighted by Mastery Development, 1991, all rights reserved. The software may not be duplicated, disassembled, loaded onto a networking device (special disks marked "Network Version" and the M. D. School Tracking Network excepted), multiloaded, or distributed by any means to simultaneously reside in more than one computer. The reports produced by the *Math Facts Tracker* program may not be duplicated by other software programs. Mastery Development grants the purchaser the privilege of duplicating and freely distributing any portion of the Teacher's Guide & User's Manual for incidental educational purposes in accordance with U. S. copyright law. If you have questions concerning this message, please call Mastery Development for clarification.

Disk Replacement Policy: Mastery Development will exchange your damaged disk for a new disk at any time. A $5.00 handling charge must be included for all exchanges after the first 90 days. Simply return the disk along with a check or purchase order (if applicable), and a brief explanation of why the disk is being returned to assist us in tracking problems. (Some problems are obvious—e.g., fingerprints, staples, folded disks—we have seen them all and will replace the disk regardless of the problem.) A new disk will be shipped within one day of receiving your damaged disk.

Trademarks: *Math Facts Tracker*™, *Mastery Arithmetic*™, *Mastery Math*™, *Keyboarding Klass*™, *Geo-Race U.S.A.*™, *Geo-Quiz U.S.A.*™, *Spellbinders*™, and Mastery Development School Tracking Network ™ are trademarks of Mastery Development. Apple® and ImageWriter® are registered trademarks of Apple Computer, Inc.

Free Demo Disk Policy: Twice yearly Mastery Development sends free demo copies of all its products to elementary schools. These demo disks contain all the features of the product disks and operate long enough for teachers to perform a thorough evaluation. Mastery Development, upon request, also sends demo disks to teachers and teacher training institutions.

The people who brought you *Math Facts Tracker*

J. Scott Harris—Research, Software Design, Teacher's Guide & User's Manual
Scott is the Vice President in charge of research and development. He enjoys researching how children learn and designing software which tracks and reports their progress. He hates writing manuals and will often take more time to write them than the programmers take to write the program.

Peter Jensen

Game Design, Game and Systems Programming, Project Management
Peter is the Senior Programmer and Project Manager. He excels in programming animation, sound, and operating systems for Apple II computers. He is an expert assembly language programmer who enjoys finding new ways to enhance the performance of Apple II computers.

Stephen Jensen

Skills Programming, Reports Programming, Menus, Data Management
Stephen excels in handling data, and programming menus, input screens, and reports. He does all his work in assembly language in order to make the program fast and to squeeze as much program and data onto a disk as possible. He enjoys finding new methods to make a program easier to use and takes pride in the placement of every pixel for both screen displays and printed reports.

CONTENTS

1. Math Facts: Definitions and Terms 1
2. Importance of Understanding, Practicing, and Mastering the Math Facts 2
    - Efficiency 3
    - Automaticity 3
    - Exploration 4
    - Estimation 5
    - Confidence 6
3. How Children Learn Math Facts 8
    - Counting Strategies 9
        - Counting Strategies for Addition 10
        - Counting Strategies for Subtraction 11
        - Counting Strategies for Multiplication 11
    - Relational Strategies 12
        - Relational Strategies for Addition 13
        - Relational Strategies for Subtraction 14
        - Relational Strategies for Multiplication 15
        - Relational Strategies for Division 16
    - Automatization/Recall 17
4. How to Teach Math Facts 18
        1. Allow children to start at the counting stage 19
        2. Allow children to progress from one strategy to the next at their own pace 19
        3. Assist children in discovering how other children solve math facts 19
        4. Encourage exploration 20
        5. Teach strategies through direct instruction, games, and activities 20
        6. Teach alternative memorization techniques 21
        7. Provide lots of timed and untimed practice 22
        8. Don't stop until mastery is obtained 22
5. Design Features of Math Facts Tracker 23
    - Levels 23
    - Mastery Requirements 24
    - Review Problems 25
    - Individual Problem Pool 25
    - Time Requirements 25

6. Getting Started With Math Facts Tracker 27
1. Add Students 27
ADD STUDENTS—Floppy Disk Version 27
ADD STUDENTS—M. D. Network Version 27
2. Print Student List 28
PRINT STUDENT LIST—Floppy Disk Version: 28
PRINT STUDENT LIST—M. D. Network Version: 28
3. Set Password (floppy disk version only) 28
SET PASSWORD—Floppy Disk Version 28
4. Student Practice 28
5. Print Reports 29
6. Change Game Selections 29
7. Games 30
Game Design 30
Changing Game Assignments 30
CHANGE GAME ASSIGNMENTS—Floppy Disk Version 30
CHANGE GAME ASSIGNMENTS—M. D. Network Version 31
Turning Off the Games 31
Turning Off the Sound 31
Game Hints 32
Grand Prix 32
Labyrinth 32
Bomber II 32
Hall of Fame 32
Viewing the Hall of Fame 33
VIEWING THE HALL OF FAME—Floppy Disk Version 33
VIEWING THE HALL OF FAME—M. D. Network Version 33
Printing the Hall of Fame 33
PRINTING THE HALL OF FAME—Floppy Disk Version 33
PRINTING THE HALL OF FAME—M. D. Network Version 33
Resetting the Hall of Fame 34
RESETTING THE HALL OF FAME—Floppy Disk Version 34
RESETTING THE HALL OF FAME—M. D. Network Version 34
8. Reports 35
Individual Progress Report (or Parent's Report) 35
Class Summary Report (or Teacher's Report) 35
Custom Practice Pages 37
100-Fact Tests 37
Printing Reports 37
Use ImageWriter Printers 37
Printing Reports—Floppy Disk Version 37
Printing Reports—M. D. Network Version 38

9. Commonly Asked Questions 39
What Are the Computer and Memory Requirements? 39
What Are the Printer Requirements? 39
How Many Students Can Be Tracked? 39
How Do I Get To the Tracking Menu? 40
How Do I Get To the Tracking/Reports Menu? 40
How Do I Change the Password? 42
How Do I Add a Student? 42
How Do I Delete a Student? 42
How Do I Remove Naughty Names From the Student List and the Hall of Fame? 43
How Do I Change a Student's Name, Grade, or Group? 43
How Many Problems Are Presented in Each Session? 43
How Can Students Most Quickly Advance To the Next Level? 44
What Is the Mastery Development School Tracking Network System? 44
References 45
Citations 45
Other Helpful References 47
APPENDIX 49

NORTH PARK MEDIA CENTER
COLUMBIA HEIGHTS SCHOOLS

TABLES AND FIGURES

Figure 1.1 Math fact terms 1
Figure 3.1 Counting strategies for addition 10
Figure 5.1 Math fact labeled with attempt number 24
Figure 5.2 Mastery and base-line time indicators 26
Table 5.3 Effects of slow responses on the mastery time limit 26
Figure 7.1 Changing the game assigned to subtraction from Labyrinth to Bomber II 31
Figure 8.1 Individual Progress report and Class Summary report 36
Figure 9.1 Selecting the "Tracking Menu" from the Main Menu 40
Figure 9.2 The Tracking Menu with the "Add Password" option selected 40
Figure 9.3 Selecting the "Tracking/Reports Menus" option on a student station computer 41
Figure 9.4 Selecting the "Math Facts Tracker Tracking/Reports Menu" option 41

1

MATH FACTS: DEFINITIONS AND TERMS

Math facts (also known as basic number facts, times tables, addition tables, etc.) are comprised of all the single-digit combinations of the four basic operations: addition, subtraction, multiplication, and division. The single-digit rule applies to both addends for addition, the minuend and difference for subtraction, the multiplicand and multiplier for multiplication, and the divisor and quotient for division. (See Figure 1.1) The remaining terms of each operation (i.e., sum, minuend, product, and dividend) result in either one- or two-digit numbers.

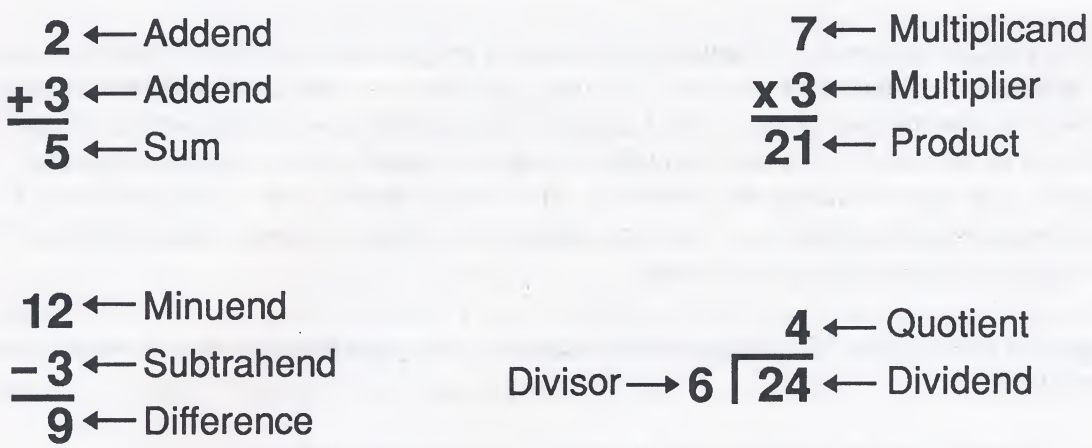


Figure 1. 1 Math Fact Terms

There are 100 math facts for each operation except for division which, because division by zero is undefined, has only 90 facts. This makes a total of 390 math facts. These 390 math facts serve as the basic building blocks for operations with larger numbers, operations with other types of numbers (i.e., decimals, fractions, etc.) and more advanced operations (i.e., roots, powers, and other functions). **The *Math Facts Tracker* program focuses on only the 390 math facts.** As the 390 math facts are mastered, students should move on to other activities involving larger numbers, decimals numbers, and fractions, returning periodically to the *Math Facts Tracker* program for a tune-up or review.

2

Importance of Understanding, Practicing, and Mastering the Math Facts

The math facts are some of the first arithmetic operations taught in school. The ability to perform math facts quickly and effortlessly is the single most important predictor of a child's future success in mathematics. The math facts are the basic building blocks of all computational algorithms. They are the tools with which many mathematical relations are first introduced. They form much of the vocabulary essential to the study of the mathematical grammar. And, because the mastery of such facts is both easily defined and measured, they are usually the proving grounds upon which students form their initial impressions of success or failure in mathematics.

In today's world of inexpensive computing devices, many people argue it is not important to spend so much time mastering the basic math facts. This argument is not new. Even before calculators were available, some people would argue that a student who exhibited continual difficulty with the math facts should be allowed to use a math fact table in order to "get on" with the process of learning arithmetic. Although calculators are wonderful, effort-saving devices with an important role to play in the elementary math curriculum, it is a great disservice to stop a student short of mastering the math facts just because using a calculator is so easy.

Here are five reasons why all children should understand the math facts and should practice them until mastered:

1. Mental recall is the most efficient method of performing math facts.
2. Automaticity of math facts enables students to focus their attention on the solution process of more complex mathematical problems.
3. Working with math facts gives students an excellent opportunity to explore mathematical principles and relationships.
4. Mastery of math facts enhances students' estimation skills.
5. Mastery of math facts boosts children's confidence in their mathematical abilities.

The remainder of this chapter will more fully explain these five reasons.

Efficiency

A person using a calculator can perform everyday calculations much more efficiently than someone doing them in his head. That is why professionals use calculators. That is why students should use calculators. But when it comes to math facts, using a calculator is one of the *least* efficient methods. In fact, even the old technology—using a fact table—is much faster than keying the problem on a calculator. Students are always incredulous to discover this fact. This experiment is always great fun: divide the class in half. One half uses calculators on a 100-fact test; the other half performs the calculations mentally. The mental group (provided it has achieved at least a modest level of competency) will always complete the 100-fact test before the calculator group is even half finished. In fact, many in the calculator group will attempt to "cheat" by not keying the facts they already know. Moreover, the students doing the problems in their heads usually produce fewer errors. Little fingers busily pressing 400 or more keystrokes have a tendency to slip quite often.

Automaticity

Students who have mastered their basic facts can do them automatically. That is, they can do them in their heads, instantly, with little or no effort. This ability gives them an advantage when it comes to solving complex, multi-step problems or practical applications which rely on math facts for their solution. Since these students can perform math facts with little or no effort, they can direct their full attention and cognitive resources to solving the larger problem.

Contrast this with students who must rely on a calculator or written table to solve their basic math facts. Not only are these methods slower, they are also much more distracting. Students who must constantly divert their attention from solving a story problem to make sure they are pressing the correct keys, often forget either important steps or their overall plan of attack. In other words, they spend so much effort on the details that they lose sight of the big picture.

Psychologists studying this phenomenon refer to it as "automatization of subskills" (Salisbury, 1990; Schneider & Shiffrin, 1977). They contend that when sub-skills are over-learned to the point of automatization, they take up little cognitive resources (i.e., attention, short-term memory, etc.), thus allowing more cognitive resources to be allocated to the main task. Studies have shown that one of the major differences between expert performance and novice performance is that automatization of sub-skills allows the experts to remain focused on the problem.

Students who have over-learned their math facts to the point of automaticity not only can perform better on multi-step mathematical tasks (Tait, Hartley, & Anderson, 1973), they are also more likely to get the big picture and enjoy the experience. Making this point by way of analogy, consider the plight of two translators—one a novice and one an expert—whose task it is to translate a passage of poetry from Russian to English. The novice translator must interrupt his translation in order to look up every third or forth word in a dictionary. These interruptions might cause the novice to lose sight of the meaning, the rhythm, or even the simple grammatical constructs of the passage. On the other hand, the expert who has complete command of the vocabulary, will be able to focus his whole attention on the meaning and beauty of the passage. Translation, for the novice, takes much time and effort and can be drudgery. Translation, for the expert, while sometimes challenging, can be enjoyable. Notice here that the expert was successful because the vocabulary was in his head and immediately accessible. Not even a hand-held electronic foreign-language dictionary can substitute for immediate, tip-of-the-brain knowledge.

Dr. Edward Effros, thirty years a professor of mathematics at the University of California at Los Angeles, states, "mathematics is above all a language. Motivation in any language comes with fluency—the power to express new ideas. But, if students do not have an immediate command of the arithmetical and algebraic vocabulary, calculations and formulas become gibberish. Having taught calculus for nearly 30 years, I can state categorically that if students can't factor $x^2 - 9$ *instantly*, it is extremely unlikely that they will pass the course. To succeed in calculus, students must be fluent in algebra—and that means the algebraic language must be in their heads, not just in their calculators" (Effros, 1989).

Now, while it is true that this manual is directed to teachers of arithmetic and not teachers of algebra or calculus, the point is still the same: the basic elements of the mathematical language must be learned to the point of automaticity. There are no elements of mathematics more basic than the basic math facts. Mastering the basic facts to the point of automaticity will make all future arithmetic exercises easier, more understandable, and more enjoyable for students.

(For an overview of automaticity from the information processing point of view and how it relates to the teaching of mathematics, see Resnick and Ford, 1981.)

Exploration

Math facts provide a great playground for children to explore the number system and arithmetic relations. This playground has well-defined borders to keep children from getting lost. It has strict rules to keep the play from degenerating into an argument about who's right and who's wrong. And it is filled with hundreds of relationships which can be combined in a thousand ways to provide the correct answers.

It is by exploration in the math facts playground that many children get their first feeling for important arithmetic relations. They learn that addition "makes more,"and subtraction "makes less." That multiplication "makes more even faster" than addition and division "makes less even faster" than subtraction. They learn that zero has no effect in addition and subtraction, just as the number one has no effect in multiplication and division (Identity Property). They learn that subtraction "undoes" addition, and division "undoes" multiplication (Inverse Operations). They learn that switching the numbers has no effect in addition and multiplication (Commutative Property).

But most importantly, with a little exploration children learn there are dozens of ways to come up with the correct answer. Some learn addition can be performed by starting with the highest number and counting onwards, rather than by counting both numbers. Some learn that addition by 9 is as simple as "subtracting one from the other number and then making it a teen." Some learn that multiplication by six is as easy as multiplying by five (a ready-known fact) and then adding the multiplier.

There are dozens of methods like these which provide "shortcuts" to students learning their math facts. Researchers refer to these methods as relational strategies. (More on relational strategies appears in the chapter "How Children Learn Math Facts.") Each strategy is based on arithmetic principles which students become familiar with as they use them. Children can, and should, be taught many of the strategies directly. But, if given enough time to explore, children will develop many of the strategies on their own. Students should be allowed plenty of time to explore the math facts, instead of being pushed directly into learning them by rote. Students given time to explore will emerge with a much better understanding of arithmetic concepts.

Estimation

One of the most important, yet difficult, skills for students to learn is estimation. For the past twenty years the National Council of Teachers of Mathematics has placed great emphasis on teaching students estimation skills. All students should be able to look at a problem and give a "ballpark answer" or determine when a calculated answer is obviously wrong. Moreover, the ability to estimate answers gives students an advantage in solving complex problems by enabling them to mentally separate the calculation portion of a problem from the higher-order thinking processes needed to solve the problem.

Students who have learned math fact strategies through exploration, and those who have mastered the math facts to the point of automaticity, have a great advantage in learning estimation skills. For example, a student familiar with the math fact strategy of splitting a problem (i.e., 6×7 is the same as 5×7 plus 7 more) will more easily learn the estimation technique of doing this with larger numbers (i.e., 35×21 is close to $(30 \times 20) + (5 \times 20)$). Another example is that of checking a computed answer to see if it is reasonable. If after adding 829 and 734 on a calculator, the first two digits should be close to 15 (since 8 + 7 makes 15). If the calculator shows 8435, a student should know instantly that something has gone wrong. In order to efficiently perform this type of answer-checking estimations, a student must be able to recall math facts with little or no effort. If the student is not proficient with math facts, the checking procedure becomes tedious and is often skipped.

Many other examples could be shown to demonstrate that familiarity with arithmetic relations and the mastery of math facts are essential prerequisite skills to learning estimation techniques. Students who do not have mastery of the math facts, or have not spent much time exploring math fact relationships, will rarely become proficient estimators.

Confidence

Student confidence in their mathematical abilities increase as they begin to understand and use relational strategies and as they master the math facts. This increase in confidence manifests itself in a more mature perception about the nature of mathematics and how it ought to be learned, in an increased level of perseverance in solving complex problems, and in increased self-esteem in how they feel about their individual math performance.

Many students have the mistaken impression that math is a monolithic discipline governed by inflexible rules. They approach math as a subject that is to be learned by memorizing the rules and algorithms, a subject where creativity has no place. But students who use relational strategies to solve math facts realize that there are many ways to solve a problem. Students who explore math facts and discover strategies on their own realize that math is a creative subject. For these students math becomes more than just a high-pressure exercise of quickly producing a correct answer—a game where those who answer immediately and correctly are smart, and those who choose to think about a problem and fuss with it a while, are dumb. Studying math fact strategies helps to mature a person's perception of mathematics and builds confidence in a student's ability to find creative solutions.

Students who gain mastery of the math facts to the point of automaticity, where recall is instant and effortless, experience more success with solving complex problems and practical applications. With instant command of the math facts, students are able to spend more of their cognitive resources focused on the problem at hand rather than fiddling with computations, whether in their heads or on a machine. This increased success, coupled with the fact that solutions are less tedious when math facts are memorized, increases the amount of perseverance a student employs when faced with complex problems.

Math facts are easily measured. Students know where they stand. Unlike some topics where students don't even have a clue as to what they still don't know, when it comes to math facts, students know if they know them—and know if they don't. Students often base their confidence in their mathematic abilities (justly or unjustly) on their confidence with the math facts. Moreover, society also judges a person on proficiency with math facts. A person who can't perform math facts is considered mathematically illiterate, just as a person who can't read is considered illiterate in a more general sense. Finally, giving a calculator to a person unable to add six and eight will do no more to boost this person's self-image (although it will help his calculation abilities enormously) than giving a tape player to a person who can't read.

In summary, studying and mastering the math facts help students in many ways. It makes all future work with math easier and less tedious. It helps students gain insights into mathematical relationships, estimation skills, and the creative nature of mathematics. It boosts students' self-confidence in math. And it enables students to factor $x^2 - 9$ instantly, thus making them a candidate to pass Dr. Effros's calculus class. Don't cheat your students by allowing them to get by without understanding and mastering the math facts.

3

How Children Learn Math Facts

Children are all different. They arrive at school from different backgrounds, with different ways of looking at things, and different ways of solving problems. A kindergarten student arrives at school on the first day equipped with significant mathematical knowledge and a set of tools to be used in solving mathematical problems (Baroody, 1987). These tools might not be the same as his neighbor's tools, and some of the tools might be faulty, but the student will be eager to use them just the same. Therefore, it should come as no surprise that even a subject as well-defined as math facts is approached differently by all children, even children receiving the same instruction.

Children employ many types of procedures and strategies to solve math facts. They learn these procedures from direct instruction, from observing others, and—most importantly—from self-discovery. Some procedures and strategies are easy to learn but are slow and inefficient. Other strategies are much more efficient but require a mature understanding of the mathematical operations and other mathematical relationships.

The procedures and strategies students use to solve math facts can usually be grouped into three categories: counting strategies, relational strategies, and automatization/recall. Counting strategies use objects, fingers, and mental counting processes and sometimes as many as three steps to compute sums, differences, and products. Relational strategies involve the use of simple mathematical relations to convert a problem into a fact already known. Automatization/Recall relies on retrieval of answers from long-term memory or the over-learning (automatization) of relational strategies to the point that they are used quickly and effortlessly.

Children usually progress from counting strategies to relational strategies and finally to automatization/recall (Siegler & Shrager, 1983). However, this progression is not universal for all children or for all facts learned by a single child (Svenson & Sjoberg, 1982, 1983). Children will progress from counting directly to recall for many of the math facts, especially the doubles (i.e., 2 + 2, 3 + 3, etc.). Many students will shift back and forth between counting strategies and relational strategies or relational strategies and recall. Many students will apply a certain strategy to one problem but not to another closely related problem. When under pressure, or when out of practice, children are likely to fall back onto the more primitive strategies which are less efficient but more understandable (Svenson & Sjoberg, 1983; Klum, 1985). But for the most part, children start with counting strategies, and then progress to relational strategies and automatization/recall in an attempt to reduce the time and cognitive processing necessary to solve each problem.

Work within each of the three categories provides benefits for the child. None of the categories should be skipped. Counting strategies help children understand the meaning and usage of arithmetic operations, since they are the most closely associated with the definition of each operation. Relational strategies help students learn important mathematical concepts and relations and, when self-discovered, are sources of great pride and confidence. Automatization/Recall is the final goal, since it is the most efficient method of generating answers and can be used without encumbering memory or attention with counting or intermediate steps. However, children should not be forced to memorize math facts right from the beginning. As children work with the math facts, they will memorize them on their own as they see it is easier to memorize them than to continue working with counting strategies or relational strategies.

The remainder of this chapter defines each of the three categories and then lists and explains the most commonly used strategies within each category. The appendix contains Strategy Usage matrices which show the facts which can be solved by each of the relational strategies, and a Dominant Strategy matrix which displays the strategies most likely to be used.

Counting Strategies

Counting is one of the first arithmetic skills learned by children and is the first tool employed by children to solve math facts. Counting strategies are solution methods which rely on counting procedures with concrete objects, fingers, or mental counts. Counting strategies are closely related to the definition of each operation (i.e., *addition*, the union of two sets; *subtraction*, the separation or partitioning of a set; etc.) and thus help children understand the terms and meaning of an operation and the appropriate situations in which the operation should be used.

Counting strategies exist for each of the operations except division. (Counting strategies could be invented for division, but they would be extremely inefficient and not likely to be used by a child cognitively ready to begin division.) Some strategies are more efficient that others. Children begin with the strategy easiest to learn and then will move on to the more-sophisticated strategies which take fewer steps and require less counting (Svenson & Sjoberg, 1982). Many children discover the more efficient counting strategies on their own without ever being taught (Fuson & Willis, 1988).

Eventually children will abandon counting strategies altogether as they move on to memorization or the more efficient relational strategies. However, since children readily understand how counting strategies work and place a high degree of confidence in their results, they will often revert back to them when under pressure or unsure of the results of other strategies (Svenson & Sjoberg, 1983; Klum, 1985).

Counting Strategies for Addition

A. COUNT-ALL: The "count-all" strategy is the first strategy adopted by children when learning to add. It is employed when a child starts with the number one and counts both addends. This strategy follows naturally from the manipulative exercise of combining two groups of objects into a single group and then counting each object. A child using the count-all strategy to add 3 + 5 will start with one and count to three using both his fingers and a verbal count. The child will then count the second addend starting over again on his fingers but continuing on with his verbal count. When the fingers reach the second addend (5) the verbal count is the sum of the two addends.

B. COUNT-ON-FROM-FIRST: Children begin to use the "count-on-from-first" strategy when they recognize it is not necessary to count the first addend. In adding 3 + 5 a child simply states the first addend (3) and then counts on from that point. The second addend is enumerated with fingers, and when reached, the verbal count is the sum of the two addends.

C. COUNT-ON-FROM-LARGEST: The "count-on-from-largest" strategy is used when the student realizes that the order of the addends does not affect the sum (commutative property) and that it is easier to start with the largest addend since this results in fewer counting steps. In adding 3 + 5 a child states the largest addend (5) and then counts on verbally while enumerating the smallest addend with fingers.

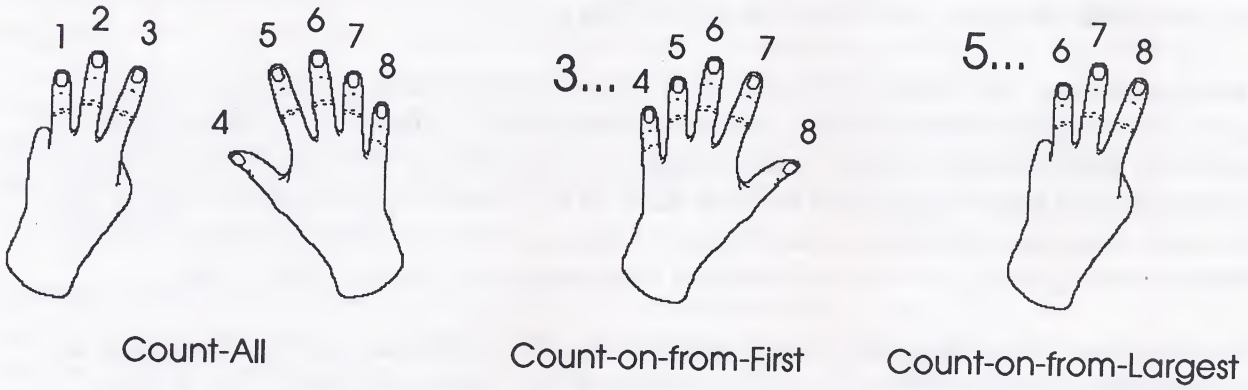


Figure 3.1. Adding 3 + 5 with counting strategies for addition.

Counting Strategies for Subtraction

A. TAKE-AWAY: The "take-away" strategy usually is the first procedure children use when starting to subtract. This strategy follows naturally from the manipulative exercise of subtracting by separating objects from a group and counting the remainder. A child employing the take-away strategy will count up to the minuend (first number) on his fingers, then, restarting his verbal count from one, will count up to the subtrahend (second number), putting fingers down as he goes. The remaining fingers represent the difference, which is then counted. As you can see, this strategy requires three separate counts and is difficult to use with numbers larger than ten.

B. COUNT-DOWN: The "count-down" strategy is used when the child realizes that counting backwards from the subtrahend will yield the difference. A child simply states the subtrahend and then mentally or verbally counts backwards from that point while keeping track of the minuend on fingers. This strategy requires two less counting steps than the take-away procedure, but requires the ability to count backwards.

C. COUNT-UP: The "count-up" strategy is easier to use than the count-down strategy when the subtrahend is close to the minuend. Children using this strategy start with the subtrahend and count up until they reach the minuend, keeping track of the count with their fingers. The final count is the difference.

Counting Strategies for Multiplication

A. REPEATED-ADDITION: Because multiplication generally is introduced as a process of "repeated-addition," this usually is the child's first multiplication strategy. A child using repeated addition to multiply 4 x 5 will add four, five times. This addition might be performed on paper or done mentally.

B. ARRAY-COUNTING: An array is a row or column of objects. In "array-counting" children make rows of objects and then count the objects. The simplest way to implement array counting is to draw horizontal rows for the multiplicand and intersecting vertical rows for the multiplier, then count the intersecting points to determine the product. Array counting is a simpler procedure than repeated addition but is also more time consuming.

C. SKIP-COUNTING: "Skip-counting" is more efficient than repeated-addition or array-counting. If a child knows how to skip-count with a certain number, he will use this method instead of repeated-addition. Usually children are taught to skip-count by twos, fives, and tens. Teaching children to skip count with all of the single digits will enable them to use the skip-counting strategy for all the multiplication facts.

Relational Strategies

Even the most efficient counting strategies are slow, tedious and error prone. In order to avoid the laborious work of counting, children begin to memorize the math facts. Complete memorization of the math facts obviously is a desired goal; however, committing 390 math facts to memory is not an easy task and usually takes years to accomplish. Fortunately, however, relational strategies can serve as a bridge between counting strategies and memorization.

Relational strategies rely on mathematical properties or relationships to convert a given math fact into an already-memorized fact. As you can see, students must have already memorized some of the math facts in order to utilize relational strategies. Fortunately, some facts are memorized very easily at an early stage. These easy-to-memorize facts include the doubles (i.e., 2 + 2, 3 + 3, etc.), problems containing 10, pairs of numbers which add to ten, and certain other combinations unique to each child. Relational strategies utilize these memorized facts to obtain the solution to the other math facts.

Besides being an important bridge between counting strategies and memorization, working with relational strategies is a wonderful way for children to explore the number system and to learn many of the mathematical concepts and relationships they will need to know for work beyond arithmetic. For example, it is through relational strategies that most children are first introduced to the commutative and associative properties. Also, the sharing strategy helps children understand the idea of inverse operations, and the "double +1 / -1" strategy and compensation strategy help children to learn the principle of compensation, which is used to balance equations.

Another benefit of studying relational strategies is that they assist in the memorization process to make the recall of facts more permanent. Children who use relational strategies, especially the more mature strategies, tend to retain their knowledge of the basic facts over summer vacation better than students who relied memorization alone (Rathmall, 1978). This is because relational strategies enable students to organize and understand relations among the facts that aid in memorization and recall.

However, the greatest benefit of relational strategies is the outlook on mathematics that children gain when they learn, discover, and use these strategies. Discovering and using relational strategies help children understand there are many methods to solve a mathematical problem and that if a certain problem appears to be too difficult, it can often be transformed into a simpler one. Working with relational strategies helps children view mathematics as a flexible and dynamic subject rather than as a set of rules to be memorized and rigidly applied.

Children should not be expected to learn all the relational strategies. After all, the goal is to learn the math facts, not the strategies. However, children will adopt a particular strategy if it makes sense to them, or if they perceive a need for it. Some strategies are employed for only a short period of time, as the child either quickly commits the math fact to memory or moves on to a more efficient strategy. Other strategies might be used for many years, even into adulthood. Children will tend to move through the strategies as their needs dictate, usually moving from those which take the most time and cognitive processing to those which are more efficient. Although there exist logical paths that one could take through the relational strategies, children do not seem to take these paths. Instead, their choice of strategies seems to be based on a variety of personal, social, and cognitive reasons (Steinberg, 1985).

Relational Strategies for Addition

1. ZERO RULE FOR ADDITION (Additive Identity): 6 + 0 = 6. "Any number plus zero stays the same." This is how a child states the "zero rule for addition." This rule is easy to learn and applies to 19 of the 100 addition facts. All students, even the mentally handicapped, will discover this rule on their own and apply it rather than memorize the zero facts (Baroody, 1988).

2. ONES RULE FOR ADDITION: 6 + 1 = 7. "Any number plus one is just the next number." Like the zero rule stated above, this rule is memorized and applied rather than the nine individual facts that it is used to solve. Children learning both the zero rule for addition and the ones rule for addition can solve 38 of the 100 addition facts, or more than one third of all possible addition facts!

3. COMMUTATIVE PROPERTY: 3 + 7 = 7 + 3. The commutative property states that the order of the addends does not affect the sum. In other words, 3 + 7 = 7 + 3, and 8 + 5 = 5 + 8. Using this strategy reduces the number of addition facts to be memorized from 100 to 55.

4. DOUBLES +1 OR -1: 6 + 7 → (6 + 6) + 1. This strategy builds on the easily memorized doubles (i.e., 3 + 3, 4 + 4, etc). Once the doubles are memorized, the near-doubles can be computed by simply adding or subtracting one from the corresponding memorized double. For example, 6 + 7 can be solved by recalling 6 + 6 = 12 and adding one more, or by recalling 7 + 7 = 14 and subtracting one.

5. DOUBLES +2 OR -2: 6 + 8 → (6 + 6) + 2. This strategy is similar to the one above but applies to addends which differ by two. For example, 6 + 8 can be solved by recalling 6 + 6 = 12 and adding two, or by recalling 8 + 8 = 16 and subtracting two.

6. SHARING: 6 + 8 → 7 + 7. This strategy is also used to solve problems where the addends differ by two. The idea is to increment one addend and decrement the other to convert the problem into an equivalent doubles problem. For example, 6 + 8 can be solved by incrementing 6 to 7 and decrementing 8 to 7, thus resulting in the easily memorized doubles fact 7 + 7. Children call this strategy sharing, because they think of it as the larger addend sharing a 1 with its neighbor.

7. KNOWN FACT +1 or -1. This strategy is similar to the doubles +1/-1 strategy, but uses memorized facts other than the doubles.

8. OVER-TEN: 8 + 5 → (8 + 2) + 3. This strategy is used with the larger-number addition facts. It is employed by decomposing one of the addends (usually the smallest) into two parts. One part combines with the other addend to make ten, and the other part is added to ten to find the sum. For example, in the problem 8 + 5, the 5 is broken up into 2 and 3, and the whole problem is expressed as (8 + 2) + 3, or 10 + 3. Children must be able to easily recognize pairs of numbers which add to ten in order to use this strategy.

9. NINES TRICK FOR ADDITION: 9 + 6 → 10 + 5. This strategy also relies on sharing to convert a problem with 9 as one of the addends into an easy-to-solve tens-problem. For example, 9 + 6 is converted into 10 + 5. Children will often state this strategy as "take away one from the smaller number and then make it a teen." Children often learn this strategy from other children and will use it without understanding why it works. Hence the name "Nine's Trick."

Relational Strategies for Subtraction

1. ZERO RULE FOR SUBTRACTION (additive identity): 7 - 0 = 7. "Any number minus zero stays the same." Just like the zero rule for addition, children easily learn this rule for subtraction, which applies to 10 of the 100 subtraction facts.

2. DOUBLES RULE FOR SUBTRACTION: 7 - 7 = 0. "Any number minus itself equals zero." Children accept this rule easily, as it follows naturally from the definition of subtraction. Once this rule is learned, it is applied automatically for all doubles subtraction problems, which account for 10 of the 100 subtraction facts.

3. ONES RULE FOR SUBTRACTION: 7 - 1 = 6. "When subtracting one, just count backwards." For example, 7 - 1 equals 7...6. Children will use this rule if they are proficient at counting backwards. It applies to 10 of the 100 subtraction facts.

4. NEIGHBORS RULE: 7 - 6 = 1. "If the numbers to be subtracted are next to each other on the number line (i.e., neighbors), then the difference is one." This rule applies to 9 of the 100 subtraction facts. Learning and applying the four rules listed above will help a child solve 38 of the 100 subtraction facts, more than one-third of the total!

5. FACT FAMILIES: 11 - 7 ➞ 7 + ? = 11. All addition and subtraction facts are part of a "fact family" which has four members. The four members for the example above are: 11 - 7 = 4, 11 - 4 = 7, 4 + 7 = 11, and 7 + 4 = 11. A child not knowing the answer to a certain fact sometimes can convert the problem into one from the same fact family to which he does know the answer. Since addition is generally easier for children than subtraction, this strategy is rarely used for addition facts; when used for subtraction facts, it is taught to children as thinking of the corresponding addition problem.

6. UP-OVER-10: 14 - 8 ➞ (10 - 8) + (14 - 10). This three-step strategy is used when the minuend (14) is larger than 10. First, find the difference between the subtrahend (8) and 10; in this case it is 2. Next, find the difference between 10 and the minuend (14); in this case it is 4. Finally, add the two differences; 2 + 4 = 6. Children often think of this strategy as building up the subtrahend to ten and then going over ten to the minuend, thus it is named "up-over-10."

   Children are most likely to adopt this strategy after they have memorized all of the tens combination subtraction facts. They then use the strength of this knowledge to break up harder subtraction problems into two easier problems dealing with previously memorized tens facts.

7. DOWN-PAST-10: 13 - 5 ➞ (13 - 10) + (10 - 5). This three-step strategy is the same as the up-over-10 strategy, but it goes the other direction. First, find the difference between the minuend (13) and 10; in this case it is 3. Next, find the difference between 10 and the subtrahend (5); in this case it is 5. Finally, add the two differences; 3 + 5 = 8. Children often think of this strategy in terms of taking the minuend down to ten and then going past ten to the subtrahend; thus it is named "down-past-10."

8. COMPENSATION: 16 - 7 → (16 - 8) + 1. Sometimes children can change a difficult subtraction fact into an easy one by adding or subtracting one or two to either the minuend or the subtrahend. In the example above, 16 - 7 was converted into the easier reverse doubles problem 16 - 8 by adding one to the subtrahend (7). Once the easier problem is solved, changes to the subtrahend must be compensated by doing the same to the answer, and changes to the minuend must be compensated by doing the opposite to the answer. Children which adopt this method sometimes get confused which way to compensate the answer. This confusion can be avoided by encouraging the use of this strategy with only the most able children, or by teaching compensation with only the subtrahend.

9. NINES TRICK FOR SUBTRACTION: 16 - 9 → 17 - 10. This strategy is simply the compensation strategy applied to the tens facts and can be applied by subtracting ten and then adding one. However, children don't usually realize the relationship between the nines trick and the compensation strategy. Instead, they simply take the ones digit from the minuend and add one.

10. SCOTT'S METHOD: 13 - 5 → 5 - 3 = 2 → 10 - 2. This strategy is a two-part method which breaks up harder subtraction problems involving a borrow into two simpler problems without a borrow. First, the first digit of the minuend (3) is subtracted from the subtrahend (5). Then, this result (2) is subtracted from 10. Hence, this method converted a difficult problem into two problems, a simple problem involving single digits and a readily memorized tens fact problem. This method can also be thought of as reducing both the minuend and the subtrahend by the digit in the ones column of the minuend.

    I discovered this method when I was in the third grade, as I was trying to come up with an easier method of subtracting multi-digit numbers containing many caries. I was a very proud third-grader but never got my teacher to understand fully what I was doing. She encouraged me to continue using it, however, and named it "Scott's Method." This method is not easily taught, but it is an example of methods students will discover on their own, and this type of exploration and self-discovery should be encouraged.

Relational Strategies for Multiplication

1. ZERO RULE FOR MULTIPLICATION: 6 x 0 = 0. "Any number times zero is zero." Like the same strategy for addition, this rule is easy to learn and applies to 19 of the 100 multiplication facts.

2. ONES RULE FOR MULTIPLICATION (Multiplication Identity): 6 x 1 = 6. "Any number times one equals the same number." Like the zero rule for multiplication above, children easily learn and apply this rule rather than memorize each of the ones combinations. The ones rule and the zero rule combined account for 38 of the 100 multiplication facts.

3. COMMUTATIVE PROPERTY: 3 x 7 = 7 x 3. The commutative property states that the order of the multiplicand and the multiplier does not affect the product. Using this strategy reduces the number of multiplication facts to be memorized from 100 to 55.

4. RECALL HALF THEN DOUBLE: 4 x 7 → 2 x (2 x 7) → 2 x 14. This strategy is used when either the multiplier or the multiplicand is an even number. This strategy is employed by computing half the product (by halving one of the even numbers) and then doubling. This strategy is most often used when the half-product (i.e., 14 in the example above) when doubling does not result in a carry.

5. HALVING: 5 x 6 ➞ half of (10 x 6) ➞ half of 60 = 30. The halving strategy is used almost exclusively with the fives products. The strategy is to multiply the non-five number by ten and then take half. Children will adopt this strategy once they learn how simple it is to multiply by ten.

6. ADD OR SUBTRACT FROM A KNOWN PRODUCT: 6 x 7 ➞ 5 x 7 + 7. This strategy is the most-used strategy for multiplication. It is employed when the child can't recall the given problem but can remember the product of a problem where one of the factors is just one or two away. The product of the recalled problem is then changed by adding or subtracting the appropriate number of factors. For example, if the child can't remember 6 x 7 but can remember 5 x 7 = 35, the product is obtained by adding 7 to 35. Or, if the child can remember that 7 + 7 = 49, the product is obtained by subtracting 7 from 49. This strategy is most likely to be employed with the easily memorized fives facts or doubles acting as an anchor point.

7. NINES TRICK FOR MULTIPLICATION: 9 x 7 = 63. Children usually first learn to perform the "nines trick for multiplication" on their fingers. Hold out all ten fingers and then bend down the finger corresponding to the number being multiplied by nine. The fingers to the left of bent finger represent the tens place of the product, and the fingers to the right represent the ones place. After a while children should be taught another version of the nines trick which can be performed mentally. This is done by realizing that the tens digit is always one less than the number being multiplied by nine and that the ones digit and the tens digit added together always sum to nine. For example, in the problem 9 x 7, the tens digit is one less than 7, or 6, and the ones digit is found by determining what number plus six equals nine.

Relational Strategies for Division

1. ZERO RULE FOR DIVISION: 0 ÷ 7 = 0. "Zero divided by any number is always equals zero, and any number divided by zero is undefined." Children will often ask what "undefined" means and will often just apply the first part of the rule to all division problems containing zero. This is probably acceptable since they will not likely encounter any problems with division by zero, certainly not in any practical setting anyway. This rule applies to nine of the ninety division facts.

2. ONES RULE FOR DIVISION (multiplication identity): 7 ÷ 1 = 7. "Any number divided by one stays the same." This rule applies to ten of the ninety division facts.

3. DOUBLES RULE FOR DIVISION: 7 ÷ 7 = 1. "Any number divided by itself equals one." This rule applies to nine of the ninety division facts. By learning all three of the division rules, a child can solve thirty-six of the ninety possible division facts.

4. FACT FAMILIES: 24 ÷ 6 ➞ 6 x ? = 24. Like addition and subtraction facts, multiplication and division facts can also be grouped into fact families with four members each. The fact family for the example above is 4 x 6 = 24, 6 x 4 = 24, 24 ÷ 6 = 4, and 24 ÷ 4 = 6. If one member of the family cannot be solved but another member is readily recalled, the child should use the easily recalled member to solve the problem. Children usually think of this strategy as solving a division problem by finding the corresponding multiplication problem—usually never reverting to division to find the answer to a multiplication problem.

5. ADD OR SUBTRACT FROM A KNOWN FACT: $56 \div 8 \rightarrow 48 \div 8 = 6 \rightarrow 7$. As in multiplication, this strategy is the real workhorse for division. This strategy works by recalling another division fact with the same divisor and then using this reference point to add or subtract the divisor to get to the original dividend. In the example above, a child might not remember the answer to $56 \div 8$ but can recall that $48 \div 8 = 6$. Now, all that remains is for the child to add 8 to 48 until getting 56, incrementing the quotient (6) each time 8 is added. Subtraction works the same way except the quotient is decremented.

6. NINES TRICK FOR DIVISION: $63 \div 9 = 7$. The "nines trick" for division is to look at the tens digit from the dividend (6) and add one more. This trick will turn 10 of the most difficult division problems into some of the simplest.

Automatization/Recall

Automatization/Recall is the final stage in mastering the math facts. At the automatization/recall stage, children can produce the answer to a math fact quickly and without any apparent cognitive processing. This ability is important because it enables students to solve complex multi-step math problems where one of the steps is a math fact, without losing focus of the overall problem. Whether the problem is a story problem, a multi-digit operation, or algebraic factoring, the ability to recall math facts instantly means the student will be able focus all his attention on solving the problem without pausing to count or having to fill short-term memory with the intermediate steps of a relational strategy. Mastery of the math facts at the automatization/recall level makes all future work in mathematics quicker, easier, and more enjoyable.

The automatization/recall level derives its name from the two possible methods used by people to arrive at this level. The first method is to over-learn a relational strategy to the point it can be used quickly , automatically, and without consciously thinking about it (Steinberg, 1985; Baroody, 1983; Steinberg, Baroody, & Fuson, 1984). The second method is to commit a math fact to long-term memory and simply recall it when needed. Researchers still debate which of these two methods adults use to work math facts (Ashcraft & Battaglia, 1975; Baroody, 1985; Hamann & Ashcraft, 1985). Many of us probably use both methods. The important point, however, is that children obtain this level for a very high percentage of the math facts.

Children memorize some math facts more easily than others. Researchers have long known that in addition, children memorize the double facts (i.e., 2 + 2, 3 + 3, 4 + 4, etc.) much more easily than all other facts (Hamann & Ashcraft, 1985; Parkman & Groen, 1971, 1972; Miller *et al*, 1984). In subtraction, children easily memorize the reverse doubles (i.e., 4 - 2, 6 - 3, 8 - 4, etc.). In multiplication, children memorize the twos facts, fives facts, and tens facts. The twos facts are the same as the addition doubles. The fives facts are memorized quickly because of childrens early facility with skip-counting by five. And strictly speaking, the tens facts are not really math facts (since 10 is a two digit number), and they are probably learned by automatizing the tens rule ("Add a zero") rather than by memorization, but they are important because they are used as points of reference with other relational strategies.

4

How to Teach Math Facts

Earlier this century, math facts were taught by rote memorization of math fact tables (i.e., addition tables, multiplication tables, etc.). Not only were these tables difficult to memorize, once they were memorized they were not very useful. This is because the tables were memorized as a whole with only a few entry points and not as separate facts. This meant most students would have to recite part of the tables in order to get to the fact of interest, just as you might have to repeat some of the alphabet to determine which letter comes before another.

Students memorized the multiplication tables because their teachers told them to do so. However, they learned to perform their math facts just like children of all ages have, they used counting, relation strategies (called round-about procedures or generalizations), and memorized the individual facts. Our ideas about teaching math facts have changed since the days of filling children's heads with multiplication tables. One point that has become clear is that the use of counting strategies and relational strategies are natural—even if you don't teach strategies to your students, they will invent them and use them anyway in order to reduce the effort necessary to compute the answers demanded of them.

As discussed in the last chapter, children start learning math facts by using one of the counting strategies. As they tire of using a particular counting strategy, they progress to a more efficient counting strategy. Finally, they either memorize the fact outright, or move through one or more of the relational strategies on their way to the automatization/recall stage.

The process of mastering all 390 math facts might take six years or even more. Practice will help children to learn their math facts, but the process cannot be sped up past a child's natural limits, as children need to reach a certain level of intellectual maturity to progress from one stage to the next. However there are many things a teacher can do to help their students learn the math facts.

1. Allow children to start at the counting stage.

Children should always start at the counting stage each time a new operation is introduced. There are many benefits that come from counting. Most children are confident in their counting skills and feel most comfortable if allowed to use them when learning a new operation. Counting strategies help children understand the mathematical meaning of an operation. For example, solving addition and subtraction problems by counting helps children understand that addition is the union of two sets, and subtraction is the separation or partitioning of a set; solving multiplication problems by skip-counting helps children understand that multiplication of whole numbers is just repeated addition. As children come to understand the meaning of each operation through counting, they will be able to better determine which operation should be applied in various situations. Another benefit of allowing children to work with counting strategies is that it will give them confidence in their answers as they begin to use the more abstract relational strategies.

Even if children are third-graders being introduced to multiplication, allow them to explore the operation by counting. When a child gets tired of counting, he will look for other methods.

2. Allow children to progress from one strategy to the next at their own pace.

Children progress from the less efficient to the more efficient counting strategies and finally to one or more of the relational strategies or to direct memorization. Children use a particular strategy until they discover an easier method. But this discovery—whether by direct instruction, watching a neighbor, or by self-discovery— will not come until a child is ready. For example, a child will not add by counting from the larger addend until he understands the principle of conservation and realizes that starting with the larger number is appropriate. Also, a child will not use counting strategies which use decrements until he feels comfortable counting backwards. When it comes to the relational strategies, sometimes a child will accept a method without understanding how it works, such as the nines trick strategy for multiplication or division. However, the student will not usually accept this trick unless he has the facility to verify (through repeated-addition or another primitive method) that it works every time.

It is fruitless to force students to adopt certain strategies before they are ready. Instead of teaching children that a particular strategy is the right method, all children should be taught that there are a variety of strategies—all of which are proper to use—but that some strategies are easier to use than others, and that they as students should always be looking for the easiest strategy. All strategies will need to be presented repeatedly, since children will become ready to adopt them at different times.

3. Assist children in discovering how other children solve math facts.

Many of the math facts are learned from other children. Children are very keen on learning effort-saving strategies from other students. When you find a student using a strategy you think others might be ready to learn, encourage that student to share this strategy with the class. Praise the student for finding another way to do the problem, and encourage all the students to discover strategies on their own.

Another method of helping students share their strategies is to divide the class into small groups and challenge each group to find as many ways as possible to solve a few selected problems. A third method is to place a problem on the board and ask a student to write the answer and explain its solution. Then ask the rest of the class to raise their hands if they would use a different strategy to solve the problem. Continue until no additional strategies are volunteered, then ask if someone can discover another strategy on their own. You should repeat these activities every so often, since not all children experience the need or possess the intellectual maturity to pick up a new strategy at any given time. A child might see a strategy demonstrated many times before he is ready to adopt it.

4. Encourage exploration.

Children should be encouraged to invent strategies on their own. Some children will invent unique and wonderful strategies. Others will invent unique and not-so-wonderful strategies which are slow or don't always work. Most children will think they have invented an original strategy when they adopt one they have seen demonstrated many times but which they are just now beginning to understand. Praise and encourage all these efforts.

One of a student's proudest moments is when he/she discovers something on his/her own. Don't worry too much about faulty or inefficient strategies, for a child who knows he or she can change strategies will do so when poor strategies don't perform well. By using many types of strategies, and by inventing their own short cuts, children learn that mathematics is a flexible and dynamic subject which can be approached in many ways. Instead of just learning the "teacher's method" or the "book's method," children learn the benefits and satisfaction of exploring new methods on their own.

5. Teach strategies through direct instruction, games, and activities.

Don't be afraid to teach strategies directly, step-by-step, right on the chalkboard for everyone to see. Not every strategy has to be self-discovered. Children readily adopt strategies which are explicitly taught (Thornberg, 1978). Also, children are more likely to use and invent strategies if they know that they are understood and encouraged by the teacher. Both Thiele (1938) and Swenson (1949) found convincing evidence that direct teaching of strategies facilitates children's learning and retention of the basic addition facts and transfer to other problems. Stienberg (1985) found that students' use of relational strategies more than doubled during periods of direct instruction.

Not all students will understand strategies when formally presented, and fewer still will feel a need to adopt them. Keep teaching them, though, as each time you do, a few more students might be ready to move to a better method. Moreover, you will be surprised how many children "invent these strategies on their own" just a day or two after seeing it repeated for the umpteenth time.

Another method of teaching strategies is the use of games and activities which model certain strategies, lead children to the brink of discovery, or just give children an opportunity to work enjoyably with math facts. One such activity which models the Count-on-from-First strategy is called "Fill the Can." To perform this activity, you need a can, some counters (i.e., marbles or other small objects), and cards with numbers written on them. Start by audibly counting a certain number of counters into the can, eight for example, and then label the can by taping the eight card onto the can. Then have a child come forward and choose another card and write the addition problem on the board. The child then selects as many counters as was shown on the card, next placing the counters into the can one at a time while counting on from eight. After all the counters are in the can, the teacher writes the final count on the board, the child dumps all the counters on the table and counts them to verify the answer. Repeat this activity many times with the same number on the can. After all the children have confidence in this method, the verification count should be skipped.

Other game and activity ideas can be found in the following books and articles: *Games: Practice Activities for the Basic Facts* (Ashlock & Washbon, 1978), *"Look Ahead" Activities Spark Success in Addition and Subtraction Number-Fact Learning* (Thornton, 1989), *Matter of Facts* (Thornton & Toohey), *Math Activities for Child Involvement* (Dumas & Schminke, 1977), and *Strategies and activities—learning to add and subtract* (Klum, 1985). In addition, this author is preparing a book entitled *Helping Children with Math Facts* , which includes many game and activity ideas for parents and children to use at home.

6. Teach alternative memorization techniques.

There are ways to memorize the math facts other than plain old repetition. Songs, rhymes, and funny stories can provide children with a memory peg to hang up some of the more slippery facts. Al Kuehnast, a computer coordinator in the Poudre School District, Fort Collins, Colorado, has collected a list of rhymes and association clues which help students remember math facts. Being a computer specialist, he naturally wrote a program called "Clue Multiplication" to present these clues. Some examples of these association clues are:

- $8 \times 8 = 64$: 8 and 8 sat on the floor and then they became 64.
- $8 \times 3 = 24$: I "8" the sandwich at 3. What numbers sandwich 3? 2 and 4!
- $7 \times 7 = 49$: Some say it takes two John Elways (#7) to make a "49er". (This association clue probably works best in Al's home state—Colorado.)

In addition, Thomas Armstrong (1983), reports his success with songs, chants, participation stories, and bean-bag games in an article titled *"Chasing Away the Times Tables Blues"*

7. Provide lots of timed and untimed practice.

Both timed and untimed practice are important. Untimed practice removes the pressure of the clock and gives children an opportunity to explore new strategies and transfer the use of their best strategies to other math facts. Untimed practice should cover all the facts, but should especially emphasize recently learned facts or those which give a child the most trouble. Conditions for exploration are most ripe when a child is challenged with facts just on the edge of his ability and who is then is given time to wrestle with them. The Custom Practice Pages feature of the *Math Facts Tracker* program provides children with untimed practice exercises which follows these guidelines and which are customized to each student.

Timed practice is also important. Children are not likely to discover new strategies or to switch to more efficient strategies under the pressure of timed practice. However, research shows that timed practice will increase the speed and accuracy of whatever strategy is being employed (Brown & Chazal, 1935). Also, regular doses of timed practice will give children the necessary motivation to adopt or discover more efficient strategies. The drill provided by the *Math Facts Tracker* program was designed to provided timed practice and to customize the presentation of math facts to meet each student's ability level.

8. Don't stop until mastery is obtained.

This last step might seem obvious; however, almost all schools stop work on the math facts before most of the children have mastered all the facts. This is because most schools use the three-minute timed drill to determine mastery, but this drill cannot report mastery on individual facts. Without a comprehensive tracking program which can access how each child is doing on each of the 390 math facts, it is impossible to determine when a child has mastered all 390 facts. The *Math Facts Tracker* program is an appropriate tool for tracking each student's mastery of each of the math facts.

Keeping your students working on the math facts until mastery is obtained will guarantee them all the benefits outlined in chapter two.

5

Design Features of Math Facts Tracker

The most important design feature of the *Math Facts Tracker* program is the method it uses to select and present math facts to each student. The selection routine was designed to accomplish the following goals. First, students are assigned the easiest facts; then they are advanced to the more difficult facts as ability increases, thus building confidence. Second, to avoid overload frustration, a cap is placed on the number of problems in each student's problem pool. Third, the facts are grouped according to the most commonly used strategies, thus facilitating transfer. Fourth, the response time required for mastery is adjusted according to prior performance and is set at a level which makes the student stretch, thus gently encouraging students to move onto more efficient strategies and finally towards recall/automatization. And fifth, the facts are systematically reviewed to assure mastery.

The *Math Facts Tracker* program accomplishes these goals by dividing the 100 facts for each operation into four levels, keeping a comprehensive database for each student, and customizing all presentation, timing, and review decisions based on each student's past performance. The remainder of this chapter will provide a detailed description of the mechanisms used to customize the presentation of math facts. If you are not interested in this level of detail, you may skip to the next chapter.

Levels

All the math facts for each operation have been divided into four separate levels labeled Beginner, Intermediate, Advanced, and Expert. All students start at the Beginner level and work their way up to the Expert level. Students using the program for the very first time can gain an early advancement to the Intermediate level by successfully answering all six problems on the first screen or eleven of twelve problems on the first two screens. Otherwise, students are advanced to the next level when they have mastered all but two of the problems in their current level.

The math facts were grouped into levels based on relative difficulty and also by the strategy most likely to be used. This method of grouping provides two benefits: First, it assures students are presented with the easiest problems first, progressing to those which are more difficult. Second, by grouping the problems by strategy, students are more likely to transfer the application of a particular strategy to many other similar problems within the group. (Baroody, 1988; Steinberg, 1985; Thiele, 1938).

The appendix shows which math facts are assigned to each level and which strategies are most likely to be used for each fact. The M. F. T. Level matrix found in the appendix roughly corresponds to the many studies performed early this century to determine the relative difficulty of various math facts. (Baston & Combellick, 1925; Brownell & Carper, 1944; Clapp, 1924; Wheeler, 1939) This correspondence cannot be absolute, since the studies themselves reported different results (Brownell, 1934), having used different definitions of relative difficulty as well as different practice and testing procedures. Although the relative difficulty for any set of math facts cannot be fixed, emerging general patterns show certain groups of problems are more "difficult" than others. The grouping of math facts for *Math Facts Tracker* was based on the general patterns gleaned from the above studies and then adjusted to account for the strategies most likely to be used in solving these problems.

Mastery Requirements

The *Math Facts Tracker* program requires that each problem be answered correctly three times in succession in order for it to be considered mastered. Students will notice that each problem is labeled with either "NEW,""1st TRY","2nd TRY,""3rd TRY,"or "REVIEW." (See figure 5.1.) Problems are randomly selected from the unmastered problems in a student's current level and all the problems in a student's review pool. An unmastered problem is labeled "NEW" when it is presented for the first time during a session. If it is answered successfully (correct response and within the first time limit), it becomes an "in-progress" problem and is assigned an increased chance of being selected for presentation. When this problem is presented the second or third time, it is labeled "2nd TRY" or "3rd TRY." If the problem is successfully answered the third time, it is marked as "mastered" and enters the review pool. If at any time the problem is unsuccessfully answered, it remains as an "in-progress" problem (thus retaining its increased chance of selection over review problems or other unmastered problems in the student's level), and when presented again it will be labeled "1st TRY."

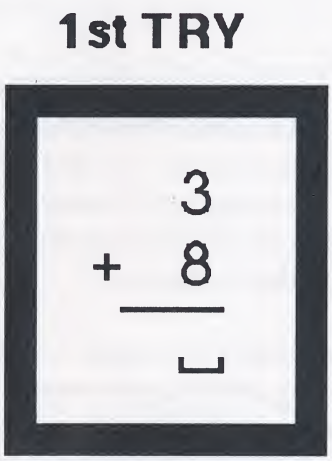


Figure 5.1 Math fact labeled with attempt number.

Review Problems

Review problems are selected from each student's pool of mastered problems. All mastered problems have a chance of being presented as review problems, but problems from a student's current level and problems which have made the "trouble facts" list are given a higher priority and are assigned a increased chance of selection. When a student successfully answers a review problem, it is thrown back into the review pool and will reappear sometime in the future. When a student responds unsuccessfully to a review problem, the problem is removed from the list of mastered facts, is marked as an "in-progress" problem (thus increasing its chances of selection), and upon reappearing is labeled "1st TRY."

Individual Problem Pool

As stated above, students can be presented with new problems, in-progress problems, or review problems. New problems automatically become in-progress problems. Review problems become in-progress problems if they are unsuccessfully answered. The in-progress problems make up a student's individual problem pool and are limited to a maximum of twelve. As the number of in-progress problems approaches twelve, a student will receive fewer new and review problems. When the number of in-progress problems equals twelve, a student will not be presented with any more new problems until some of the in-progress problems are mastered. Additional review problems are suspended when the problem pool reaches twenty. This feature helps keep students focused on a small subset of problems and reduces the frustration that would arise if the student was expected to master all one hundred facts at once.

Time Requirements

There are two time limits associated with the presentation of each math fact. The first time limit (Range: 3 to 7 seconds) is the mastery limit and is indicated by the diminishing inside bar of the presentation box (see Figure 5.2). In order for the fact to be marked as mastered, students must correctly respond to the math fact before the mastery time limit expires. Students fulfilling this requirement are rewarded with double game pieces. The second time limit (range: 27 to 63 seconds) is the base-line limit and is indicated by the diminishing middle bar of the presentation box. A correct response before the base-line time expires will earn the student a single game piece, cause the fact to be marked as "hesitate," and will reset the problems counter to "1st Try." Thus, expiration of the base-line time limit is handled in the same manner as an incorrect response.

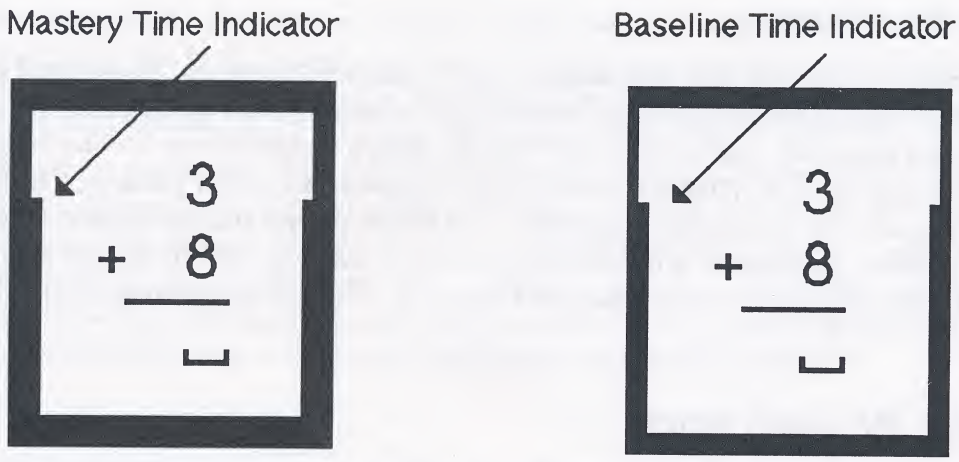


Figure 5.2 Mastery and Base Line Time Indicators

The purpose of the two time limits (and the different rewards associated with them) is to gently encourage students to discover more efficient strategies and ultimately to move to the automatization/recall stage for each fact. Since no single pair of time limits would be adequate for all students, or indeed even for a single student over the course of a year, the time limits are constantly adjusted according to student performance. New students start out with a mastery time limit of 4.5 seconds (grades one–three) or 3.5 seconds (grades four and above). At the end of each 24-problem session, the mastery time is adjusted according to the number of slow responses. (See Figure 5.3) The base-line time limit is also adjusted with student response and is roughly equal to nine times the mastery limit.

Number of Slow Responses	Change in Mastery Time Limit
0-1	- 0.125 seconds
2-3	no change
4-7	+ 0.125 seconds
8-11	+ 0.25 seconds
12+	+ 0.375 seconds

Figure 5.3 Effects of slow responses on the mastery time limit

Although the mastery time limit and the baseline time limit are constantly adjusted and stored in a student's data file, they are not printed on a student's report. Instead, a more meaningful parameter—the average response time between the presentation of each fact and the student's initial keypress—is tracked and displayed as the speed measurement on the Parent's/Individual report.

GETTING STARTED WITH MATH FACTS TRACKER

This chapter will help you get started using the *Math Facts Tracker* program. Complete steps one to four as soon as you are able. Complete steps five and six after one or two weeks. Refer to chapter 9 for assistance in getting to the various menus.

1. Add Students

The first step is to add your students by creating student data files. Student data files are used to store progress and achievement data on each of the 390 math facts. A student cannot proceed to the skills or the games until an individual data file has been created. Student data files can be created by the teacher by using the ADD STUDENT routine (floppy disk version), or the CREATE FILES routine (M. D. network version).

Create data files for all students. Make sure you spell their names correctly. If two or more students share the same name, enter a middle initial or nickname (the computer will prompt you if you try to add two students with the same name). You may create up to fifty student data files on the floppy disk version and one thousand data files on the M. D. network version. If you are using two or more floppy disks to track student progress, make sure each name appears on only one floppy disk.

ADD STUDENTS—Floppy Disk Version

Boot the disk. Select option 6, "Tracking Menu." Select option 2, "Add Student." Enter first name, last name, and grade level for up to 50 students. (Note: students may create a file for themselves by using the "New Student" option on the main menu. This procedure is discouraged, however, since it is likely to result in mistakes, mistyped names, and various naughty names. You can keep students from using the "New Student" function by filling the disk with fifty names.)

ADD STUDENTS—M. D. Network Version

Go to the Teacher's Menu on the teacher station computer. Select the option labeled "Create Files." Enter the class, teacher's initial, and student names for all students. Press the TAB key to create files for all the names entered.

NORTH PARK MEDIA CENTER
COLUMBIA HEIGHTS SCHOOLS

2. Print Student List

After creating the student files, you should print a student list. The student list shows all the names exactly as you entered them. Check this list to see if you made any typing mistakes. If so, use the "Change Student's Name/Grade" option (floppy disk version) or the "Change Files" option (M. D. network version) to correct the mistake. Once the list is correct, post it by the computers so students can refer to it if they have trouble entering or selecting their name.

PRINT STUDENT LIST—Floppy Disk Version:

From the Tracking Menu select option 7 Print Class Summary Report. Enter the grade of your students or enter "?" for all grades. Enter the date. Print the report.

PRINT STUDENT LIST—M. D. Network Version:

From the Teacher's Menu select the option labeled "Print List of Files." Enter the class number (or ? for all classes). Enter the teacher's initial (or ? for all initials). Print the report.

3. Set Password (floppy disk version only)

Next, set the password (floppy disks only; networks will already have a password assigned and should not be changed except by mutual consent of all teachers using the system). Setting a password will keep students from entering the Tracking Menu and deleting data files or other information.

SET PASSWORD—Floppy Disk Version

Go to the Tracking Menu. Select "Add/Change Password." Enter a password of up to ten letters or numbers. Choose a password that is easy to remember. **Write down the password and keep it in a secure place.** Re-enter the password to confirm there are no typing mistakes in your first entry. If the two entries match, a new password is recorded on the floppy disk.

4. Student Practice

You are now ready for students to work on the *Math Facts Tracker* program. Return to the Main Menu (floppy disk version) or select *Math Facts Tracker* from the Mastery Development Menu (M. D. network version). Allow the students to work on the program for at least fifteen minutes at a time. Encourage them to work on no more than one skill each day. The tracking system will adjust to a student's level more quickly if the student completes three to five consecutive sessions on a single skill each time the program is used.

5. Print Reports

After each student has had the opportunity to use the program two or three times, you should print a Class Summary Report and a batch of Individual Progress Reports (Parent's Reports). The Progress Reports should be sent home to the parents on a regular basis. Parents should be encouraged to work on the math facts with their children. The booklet *Helping Children With Math Facts* will assist parents with games and activity ideas.

See chapter 7 for instructions on printing reports.

6. Change Game Selections

After the students have been using the program successfully for a week or two, you might want to change the game selections. Changing the game selections will allow you to remove a game which is unpopular, move the most popular game to the skill you would like to emphasize most, or turn off the games altogether.

See chapter 8 for instructions on changing the game selections.

7

GAMES

Game Design

The Math Fact Tracker games were designed and created as a fun and challenging reward for top performance on the math skills. Students earn game pieces by correctly answering the math facts. Double game pieces are awarded if the fact is correctly answered within the mastery time limit. Game pieces enable the children to play the game longer and perform special tasks. Children enjoy playing these games, and since game pieces are awarded based on skill performance, the games always encourage them to perform at their best.

The *Math Facts Tracker* program contains three separate games: Grand Prix, Labyrinth, and Bomber II. Each game session lasts 30 to 90 seconds (about the same amount of time they spend on the skill sessions), depending on the skill of the player and the number of game pieces earned during the last skill session. At the end of each game session, the child is asked if he wants to continue with another skill session and game session. Children should attempt at least three to five sessions on the same skill at each setting.

Each game consists of ten increasingly difficult levels to challenge students of all abilities. Students begin play at the easiest level, and if they are successful at completing the game's objectives they are advanced to the next level. Children may repeat a level as many times as necessary in order to earn advancement.

Changing Game Assignments

Any of the three games Grand Prix, Labyrinth, or Bomber II can be assigned to any of the four arithmetic skills. If students find one game more enjoyable than the other two, it can be assigned to any—or all—of the skills. If a teacher finds one of the games objectionable (many teachers feel that war games such as Bomber II are inappropriate), it can be replaced by one of the other games. If teachers prefer that students don't play games, the games can be turned off by assigning "no game" to each of the skills. When "no game" is assigned to a skill, the students earns fireworks rockets instead of game pieces, and the screen displays a short fireworks exhibition after the completion of each session.

CHANGE GAME ASSIGNMENTS—Floppy Disk Version

Select "Change Game Selections" on the main menu. Enter the password if a password has been assigned. Select the skill for which you would like to change the game. Select the new game. (See figure 7.1.) Repeat the process to change the game for another skill or select "Quit" to record your changes and return to the main menu.

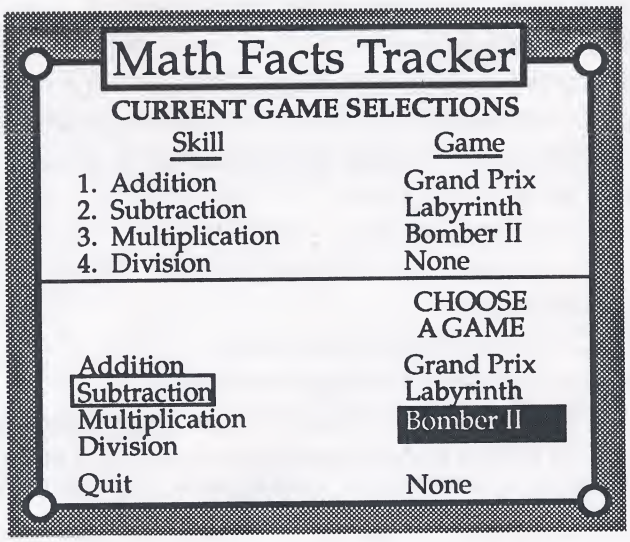


Figure 7.1. Changing the game assigned to subtraction from Labyrinth to Bomber II.

CHANGE GAME ASSIGNMENTS—M. D. Network Version

Go to the *Math Facts Tracker* Tracking/Reports Menu on either the teacher station computer or a student station computer. Select the "Change Game Selections" option from this menu. Follow the screen directions similar to those shown above for the floppy disk version.

Turning Off the Games

The games can be turned off by changing the game assignment to "No Game" for each skill (see directions on changing game assignments above). When "No Game" is selected, students are awarded fireworks rockets instead of game pieces, and a brief fireworks display is presented at the conclusion of each skill session.

Turning Off the Sound

The sound may be turned off by pressing control-S (hold the control key down, press S) from the main menu. The sound is restored when the program returns to the main menu or by re-pressing control-S. This procedure is the same for both the floppy disk version and the M. D. network version.

Game Hints

Grand Prix

- GOAL: Complete all ten race courses.
- ADVANCEMENT: Drive to the checkered winner's area.
- Driving on the edge of the track slows you down and uses more fuel.
- Drive with your car positioned vertically in the center of the screen.
- Avoid all obstacles.

Labyrinth

- GOAL: Complete all ten mazes.
- ADVANCEMENT: Collect all objects before exiting.
- Spiders become quicker and smarter at higher maze levels.
- Fire shots to stun the spiders. The more shots fired, the longer the stunning effect.
- Drop bombs to block the spiders and to clear them from a section of the maze.

Bomber II

- GOAL: Complete all ten bombing missions.
- ADVANCEMENT: Complete your mission by finding the enemy flag (hidden under a target), destroying the flag, and then landing on the right-hand runway.
- Land on right-hand runway when fuel is getting low.
- Do not bomb civilian targets!!!
- Evade enemy missiles by leading them into the ground or other objects.

Hall of Fame

Each game contains a Hall of Fame feature to keep track of the top scores. The Hall of Fame feature can be used to stimulate competition between students. In order to enter the Hall of Fame, a student must surpass his/her personal best score and also place within the top twelve scores currently recorded. If the Hall of Fame is periodically reset, students of all abilities will have a chance to enter (due to the fact that all students must surpass their personal best score, thus making it more difficult for top students to regularly fill all the slots).

The Hall of Fame can be viewed, reset, or printed at any time. These features enable the teacher to conduct regular Hall of Fame contests. Many schools using this product will conduct these contests by resetting the Hall of Fame each month, then printing and posting the results at the end of each week. The twelve students with the highest entries at the end of the month are the winners. And since it becomes increasingly difficult for the best students to continually surpass their personal best scores, even some of the slower students have a good chance of making the monthly winners list.

There are some differences in the Hall of Fame between the floppy disk version and the M. D. network version. This is because the M. D. network version can perform more extensive record-keeping than the floppy disk version. The M. D. network version keeps a separate Hall of Fame for each game and for each grade level, while the floppy disk version does not distinguish between grade levels. The M. D. network version automatically resets (erases) the Hall of Fame list at the first of each month. The floppy-disk version relies on the teacher to periodically reset the Hall of Fame list. Finally, the M. D. network version maintains an "All-Time Top-Ten" list in addition to the Hall of Fame. This list can be entered by a student of any grade level who obtains a minimum entry score (1000 for Grand Prix, 1200 for Labyrinth, 1500 for Bomber II). The All-Time Top-Ten list is a permanent list and can be reset only by the teacher.

Viewing the Hall of Fame

The Hall of Fame (and All-Time Top-Ten lists for M. D. network users) can be viewed by students or teachers at any time.

VIEWING THE HALL OF FAME—Floppy Disk Version

Press control-V (hold down the control key, press V) from the Main Menu.

VIEWING THE HALL OF FAME—M. D. Network Version

Select "View Class Hall of Fame" or "View All-Time Top 10 List" from the *Math Facts Tracker* Main Menu. The M. D. network keeps a separate Hall of Fame for each grade level. After selecting "View Class Hall of Fame" you will be asked to enter a grade level 0–9.

Printing the Hall of Fame

The Hall of Fame (and All-Time Top-Ten list for M. D. network users) can be printed on an ImageWriter printer. These printouts should be posted where students can see them.

PRINTING THE HALL OF FAME—Floppy Disk Version

Go to the Tracking Menu. Select the "Print Hall of Fame Report" option. Enter the date using the format shown on the screen (if the date had been entered earlier, this request will be skipped). Make sure the printer is online. Press any key to begin printing.

PRINTING THE HALL OF FAME—M. D. Network Version

Go to the *Math Facts Tracker* Tracking/Reports menu on either the student station computer or the teacher station computer. Select "Print Class Hall of Fame" or select "Print All-Time Top-Ten List." If you select "Print Class Hall of Fame" you will be requested to enter the class (grade 0–9 or ? for all grades).

Resetting the Hall of Fame

The Hall of Fame may be erased by the teacher at any time. A teacher might want to erase the Hall of Fame in order to start a new competition or to remove names containing bad words. See the chapter 9 to learn how to keep students from entering naughty names on the Hall of Fame list.

RESETTING THE HALL OF FAME—Floppy Disk Version

Go to the Tracking Menu. Select "Reset Hall of Fame." This feature resets the Hall of Fame lists for all three games.

RESETTING THE HALL OF FAME—M. D. Network Version

The network version keeps a separate Hall of Fame list for each class. All class Hall of Fame lists are automatically reset upon the first entry after the first day of each month. However, they can be manually reset at any time by going to the *Math Facts Tracker* Tracking/Reports Menu and selecting the "Reset Class Hall of Fame" option. Enter the class number (grade 0–9, or ? for all classes) of the lists you would like to reset. The Hall of Fame lists for all three games are reset at the same time. The All-Time Top-Ten (which is not reset automatically) is reset by going to the *Math Facts Tracker* Tracking/Reports Menu and selecting "Reset All-Time Top-Ten list."

REPORTS

The reporting capability provided by *Math Facts Tracker* is one of the key features setting this program apart from a myriad of other drill and practice programs. The *Math Facts Tracker* program collects and analyzes large quantities of performance data for each student. This performance data is used to determine which facts to present to the student. However, in order to reap the full benefits of this information, it should be regularly shared with teachers, parents, and the students themselves. Four reports have been prepared for this purpose: Individual Progress Report (or Parent's Report), Class Summary Report (or Teacher's Report), Custom Practice Pages, and 100-Fact Tests.

Individual Progress Report (or Parent's Report)

This report shows the greatest level of detail for each student. These reports should be printed and sent home with the students on a regular basis, perhaps monthly. Parents can use this report not only to see how their child is progressing, but also to see which facts their child is currently working on and which facts are giving their child the most trouble. Armed with this information, parents will be able to assist their child at home—working on the facts which need the most attention.

A special book (available from Mastery Development in January. 1992) entitled *Helping Children with Math Facts* acts as a resource guide for parents interested in working with their children at home. It explains the theory of how students learn math facts and contains game and activity ideas that help students to learn the various counting and relational strategies. Parents who use the Individual Progress Report in conjunction with this book will be able to identify which facts need to be studied and which activities can best help the students learn these facts.

Class Summary Report (or Teacher's Report)

The Class Summary Report gives the teacher an "achievement snapshot" for each student. Student achievement for all four operations is displayed with histograms. The wide bar and the number at the right shows how many facts have been mastered. The narrow bar that starts where the wide bar leaves off shows how many additional facts have been answered correctly but outside the mastery time limit. Class mean scores and median scores are displayed at the bottom.

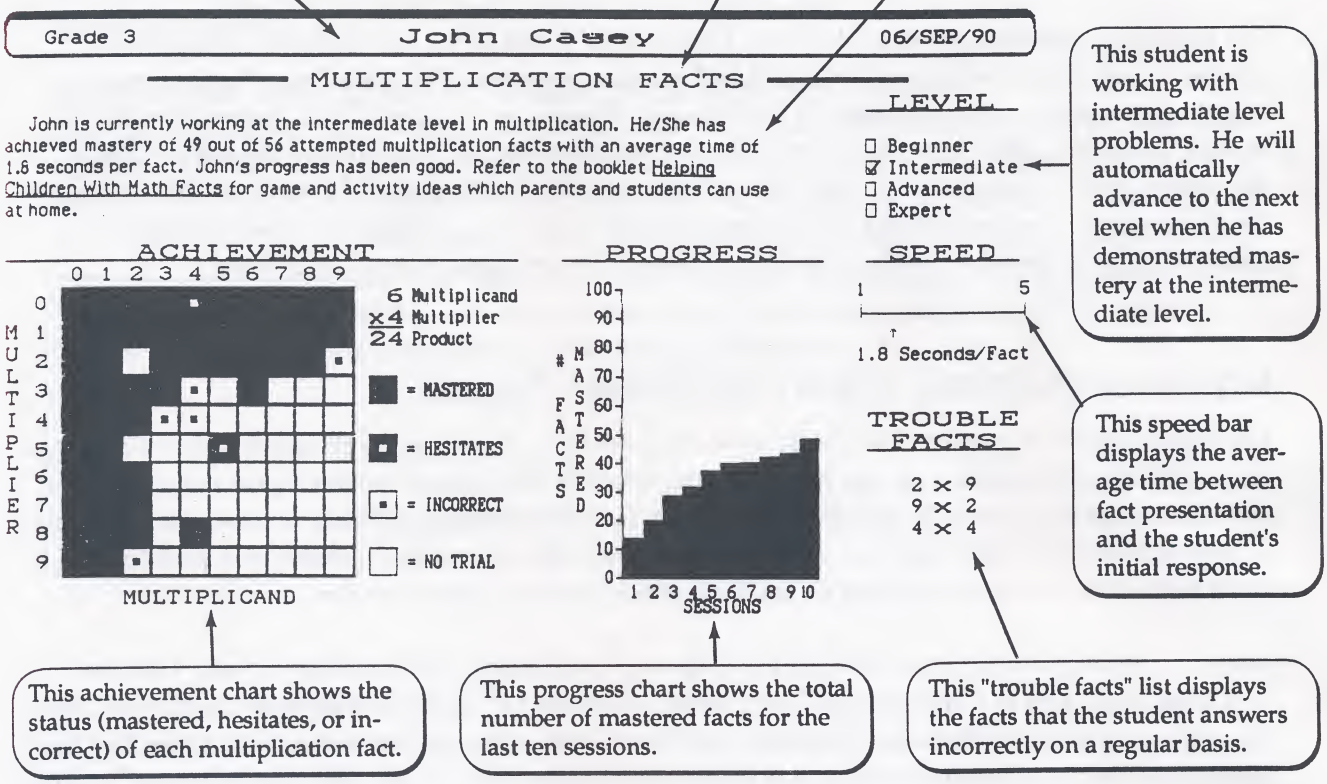
Teachers can use these reports to monitor individual and class achievement. If these reports are kept, teachers will be able to determine if a class is ahead or behind classes of past years.

Mastery Facts Tracker Parent's Report

Each student receives his/her own report. This report isspecifically designed for students to take home to parents.

This report shows progress on the 100 multiplication facts. Other reports can also be printed for addition, subtraction, and division.

This paragraph summarizes each section of the report. It is especially useful for those parents who do not wish to interpret charts and graphs.



This student is working with intermediate level problems. He will automatically advance to the next level when he has demonstrated mastery at the intermediate level.

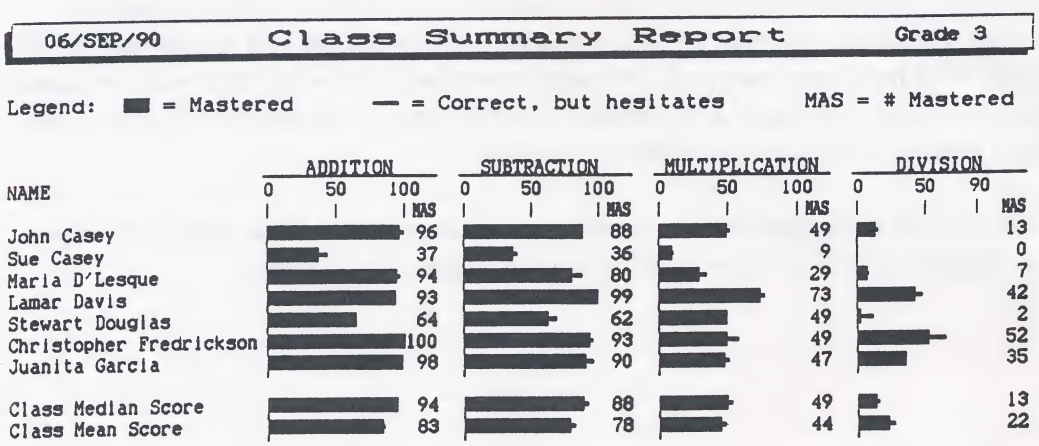
This speed bar displays the average time between fact presentation and the student's initial response.

This achievement chart shows the status (mastered, hesitates, or incorrect) of each multiplication fact.

This progress chart shows the total number of mastered facts for the last ten sessions.

This "trouble facts" list displays the facts that the student answers incorrectly on a regular basis.

Mastery Facts Tracker Class Summary Report


06/SEP/90
Class Summary Report
Grade 3
Legend: = Mastered — = Correct, but hesitates MAS = # Mastered
NAME
ADDITION
SUBTRACTION
MULTIPLICATION
DIVISION
MAS
John Casey 96 88 49 13
Sue Casey 37 36 9 0
Maria D'Lesque 94 80 29 7
Lamar Davis 93 99 73 42
Stewart Douglas 64 62 49 2
Christopher Fredrickson 100 93 49 52
Juanita Garcia 98 90 47 35
Class Median Score 94 88 49 13
Class Mean Score 83 78 44 22

This report gives the teacher an "achievement snapshot" for each student. The wide bar shows the number of facts each student has mastered. The thin bar shows additional facts which the student answered correctly but computed with an inefficient method.

Custom Practice Pages

The Custom Practice Pages report generates homework sheets for each student based upon ability and need determined from past performance. All facts from a student's current level and those from any lower level are candidates for the Custom Practice Pages report. Facts from a student's current level are given twice as much chance of being selected than those from a lower level. Facts marked "hesitate," or "incorrect,"or facts on a student's trouble list are given an even higher chance for selection. Many of the math facts, especially those with the greatest chance of selection, will be printed more than once.

Custom practice pages can contain either 50 or 100 facts (45 or 90 for division). Students on higher levels or those who have experienced difficulty with a greater number of facts will receive the longer report.

100-Fact Tests

Many schools use 100-fact three-minute drills to track student proficiency with math facts. Even though the *Math Facts Tracker* program makes this measurement tool obsolete, there are still teachers who administer this test to provide comparison data with former classes. This is why the 100-fact test feature was included on the *Math Facts Tracker* program. This option will print out all 100 math facts (90 for division) for a specific operation and place them in random order on the page. The teacher may chose to print between 1 and 99 copies of the test. All tests will contain the same 100 problems; however, the problems will be positioned differently on each test.

Printing Reports

Use ImageWriter Printers

Many of the *Math Facts Tracker* reports contain detailed charts and graphs. Not all printers have the capabilities to print these features, and other printers which could print this high level of detail would need special print driver routines which would displace room on the disk for student data files. Given these considerations, coupled with the fact that more than 82% of all printers connected to Apple II computers are Apple's ImageWriter printer, the decision was made to support only the ImageWriter I and the ImageWriter II printers. If your classroom has a different printer, you can still run the program on your computer to collect the data and then take it to another computer with an ImageWriter printer when you want to print reports.

Printing Reports—Floppy Disk Version

Boot the floppy disk on a computer connected to an ImageWriter I or ImageWriter II printer. Make sure the paper setup is correct and the printer is online. Go to the "Tracking Menu" and select the report you wish to print. Many of the reports request you to enter the group number. If you want to print reports for all students on the disk regardless of their group, enter the wildcard character "?" when asked for a group.

Printing Reports—M. D. Network Version

Reports can be printed either from a student station computer or from the teacher station computer. It is preferable to print your reports from the student station computer, since this can be accomplished without interrupting network operations. If the student station computer is connected directly to a printer, the reports will be printed on that computer's printer. If no printer is connected, the reports will be sent along the network to one of the teacher station's printers. Printing reports to a printer connected directly to a student station computer is the most efficient method of printing.

To print reports on a student station computer, select the "Tracking/Reports" menu from the Mastery Development menu. (Note: the student station computer must be an enhanced IIe or a IIgs computer to access the Tracking/Reports menu. You can distinguish between an enhanced IIe and a regular IIe by checking the computer type upon boot up. A regular IIe will be marked IIe, an enhanced IIe will be marked IIe+.) Enter the system password. Select the *Math Facts Tracker* menu. Finally, select the report you wish to print and enter all requested parameters.

To print reports from the teacher station computer, first go to the the Teacher's Menu and then select the *Math Facts Tracker* Tracking/Reports Menu. Select the report you wish to print and enter all requested parameters. When printing is completed, return to the network operation screen to enable student station computers to access the file server.

9

COMMONLY ASKED QUESTIONS

What Are the Computer and Memory Requirements?

Floppy Disk Version: Apple II+ 64k, IIe, IIc, IIc+, or IIgs. 5-1/4" disk drive required.

M. D. Network Version: Apple II+ 64k, IIe, or IIgs

What Are the Printer Requirements?

Floppy Disk Version: Apple ImageWriter I or ImageWriter II printer. Connect the printer to an Apple II+ or an Apple IIe computer with an Apple Super Serial card inserted in slot one. Connect the printer to an Apple IIc or an Apple IIgs computer by inserting the printer cable in the computer's printer port.

M.D. Network Version: Apple ImageWriter I or ImageWriter II printer. The printers may be connected either to the Teacher Station computer or directly to a Student Station computer.

How Many Students Can Be Tracked?

Floppy Disk Version: 50 students per disk, divided into 13 grade levels (K–12).

M. D. Network Version: 1000 students grouped by grade level and teacher's initial. Students can access their files from any networked computer.

How Do I Get To the Tracking Menu?

Floppy Disk Version Only: Select the "Tracking Menu" option from the "MAIN MENU" (See figure 9.1). If this is a new disk and a password has not been entered, you will be taken directly to the Tracking Menu where your first order of business should be to add a password (See figure 9.2). If a password has already been assigned, you must correctly enter the password in order to proceed to the Tracking Menu. If you cannot get past the password, ask other teachers who might have used the disk if they have changed the password without informing you. If you still have trouble, call us at Mastery Development for further assistance.

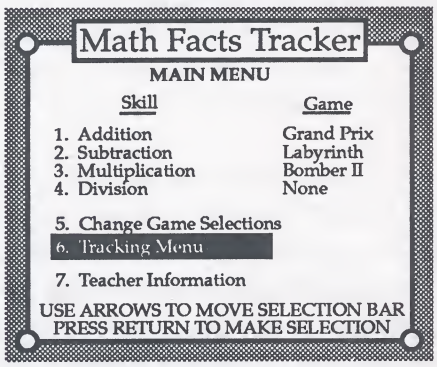


Figure 9.1.
Selecting the "Tracking Menu" from the Main Menu.

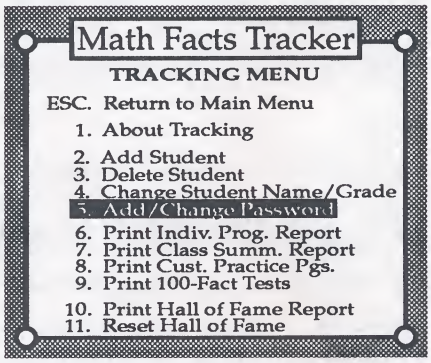

Math Facts Tracker
TRACKING MENU
ESC. Return to Main Menu
1. About Tracking
2. Add Student
3. Delete Student
4. Change Student Name/Grade
5. Add/Change Password
6. Print Indiv. Prog. Report
7. Print Class Summ. Report
8. Print Cust. Practice Pgs.
9. Print 100-Fact Tests
10. Print Hall of Fame Report
11. Reset Hall of Fame

Figure 9.2.
The Tracking Menu with the "Add Password" option selected.

How Do I Get To the Tracking/Reports Menu?

M. D. Network Version Only: The network version uses a menu called "Tracking/Reports" to print all reports, handle the Hall of Fame, and alter game assignments. (Addition and deletion of student data files is handled on the Teacher's Menu, found on the teacher station computer.) The Tracking/Reports menu appears on both the student station computers and the teacher station computer. These menus are identical, with one important exception: using the Tracking/Reports menu on a student station computer will not suspend network operations as does using this menu on the teacher station. Therefore, teachers are encouraged to print all reports using the student station version of this menu.

The *Math Facts Tracker* program is one of many Mastery Development programs with a corresponding Tracking/Reports menu. Therefore, it is commonly called the *Math Facts Tracker* Tracking/Reports Menu. To get to this menu on a student station computer, first select "Tracking/Reports Menus" from the Mastery Development menu (See Figure 9.3.), then select "Math Facts Tracker" (See Figure 9.4). To get to this menu on the teacher station computer, first go to the teacher's menu, then select "Math Facts Tracker" from the menus listed in the "Tracking/Reports Menus" section.

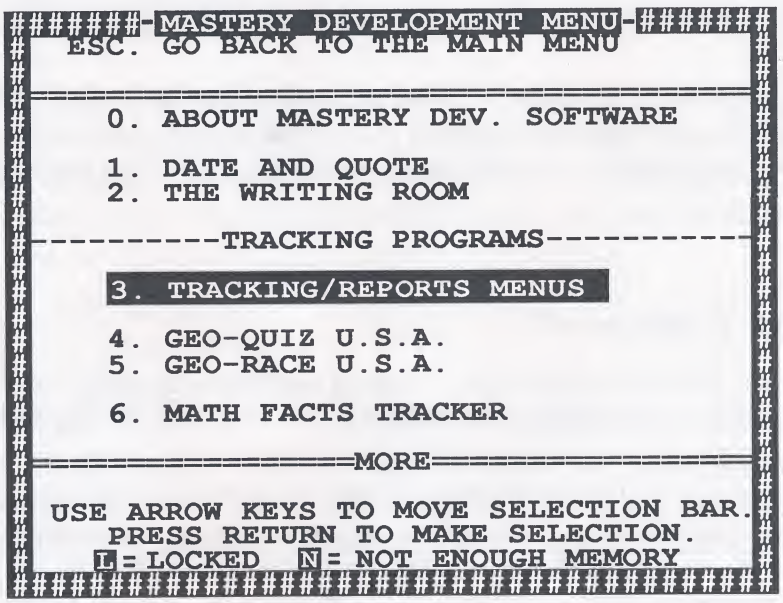

MASTERY DEVELOPMENT MENU
ESC. GO BACK TO THE MAIN MENU
0. ABOUT MASTERY DEV. SOFTWARE
1. DATE AND QUOTE
2. THE WRITING ROOM
TRACKING PROGRAMS
3. TRACKING/REPORTS MENUS
4. GEO-QUIZ U.S.A.
5. GEO-RACE U.S.A.
6. MATH FACTS TRACKER
MORE
USE ARROW KEYS TO MOVE SELECTION BAR.
PRESS RETURN TO MAKE SELECTION.
L = LOCKED N = NOT ENOUGH MEMORY

Figure 9.3. Selecting the "Tracking/Reports Menus" option from the Mastery Development Menu on a student station computer

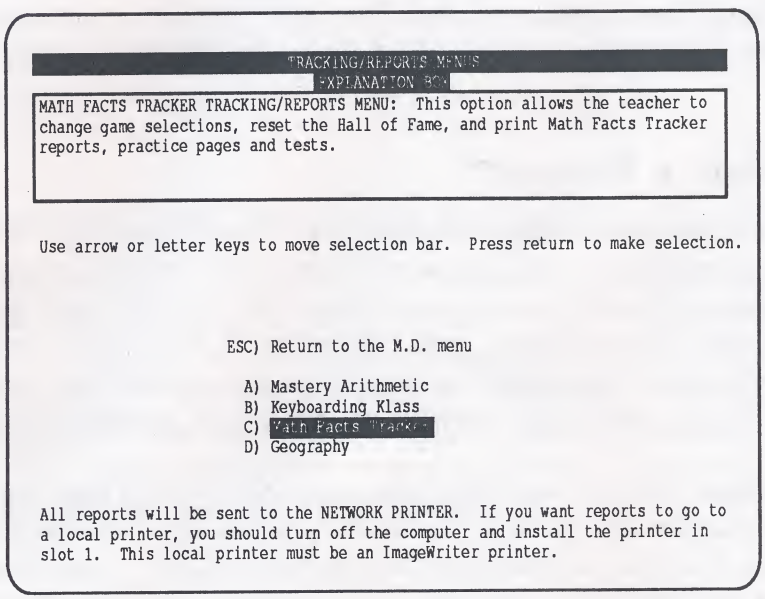

TRACKING/REPORTS MENUS
EXPLANATION BOX
MATH FACTS TRACKER TRACKING/REPORTS MENU: This option allows the teacher to change game selections, reset the Hall of Fame, and print Math Facts Tracker reports, practice pages and tests.
Use arrow or letter keys to move selection bar. Press return to make selection.
ESC) Return to the M.D. menu
A) Mastery Arithmetic
B) Keyboarding Klass
C) Math Facts Tracker
D) Geography
All reports will be sent to the NETWORK PRINTER. If you want reports to go to a local printer, you should turn off the computer and install the printer in slot 1. This local printer must be an ImageWriter printer.

Figure 9.4. Selecting the "Math Facts Tracker Tracking/Reports Menu" option on the student station computer

How Do I Change the Password?

Floppy Disk Version: Go to the Tracking Menu. Select "Add/Change Password." Enter a password of up to ten letters or numbers. Choose a password that is easy to remember. **Write the password and keep it in a secure place.** Re-enter the password to confirm there are no typing mistakes in your first entry. If the two entries match, a new password will be recorded on the floppy disk.

M. D. Network Version: Select the "Change Password" option from the Teacher's Menu on the teacher station computer. Follow the directions displayed in the explanation box at the top of the screen. Note: this password is used for the entire network system—make sure all teachers are informed of the new password.

How Do I Add a Student?

Floppy Disk Version: There are two methods of adding students on the floppy disk version. First, students can add themselves by selecting "NEW STUDENT" from the main menu after choosing a skill. Second, teachers may add new students by selecting the "ADD STUDENT" option from the Tracking Menu. The person adding a student will be asked to enter the first name, last name, and grade level (K–12). The grade level is used to assign the initial mastery time values (see Chapter 5) and to group the students for reports. It is suggested that the teacher (or an aide) use the Add Student option to add all the students before they begin work on the disk. This will reduce the chance of errors (mistyped names, last name substituted for first name, etc.) generated when students use the New Student feature.

M. D. Network Version: Teachers may add students by creating a basic skills data file. This data file is used by all of the M. D. basic skills programs, (i.e., Mastery Math, Mastery Arithmetic, Geo-Race, Geo-Quiz, Keyboarding Klass, Spellbinders, etc.) Basic skills data files are created by using the "Create Files" routine found on the "Teacher's Menu" on the Teacher Station computer.

How Do I Delete a Student?

Floppy Disk Version: Go to the Tracking Menu. Select the "Delete Student" option. Arrow down to place the selection bar on the student file you would like to delete. Press the return key to a select the student file for deletion. Select all files you would like to delete. A check mark will appear next to all files selected for deletion. (If you accidently select the wrong file for deletion, you can deselect the file by repressing the return key.) Once all files are properly marked for deletion, start the deletion process by pressing the tab key, or, press escape to exit the deletion process without deleting any files.

M. D. Network Version: Teachers may delete students by deleting their basic skills data file. This data file is actually used by all of the M. D. basic skills programs (i.e., Mastery Math, Mastery Arithmetic, Geo-Race, Geo-Quiz, Keyboarding Klass, Spellbinders, etc.) It should not be deleted unless you want to discard data from all the M.D. basic skills programs. Basic skills data files are deleted by using the "Delete Files" routine found on the "Teacher's Menu" on the teacher station computer.

How Do I Remove Naughty Names From the Student List and the Hall of Fame?

Floppy Disk Version Only. Naughty names are added to the student list when a student uses the "NEW STUDENT" routine to create a data file and enters a bad word for the first name or the last name. Naughty names are added to the Hall of Fame when this data file is used to play the games. Naughty names can be erased from the student list by deleting the data file. Naughty names can be removed from the Hall of fame by resetting the Hall of Fame (See section 6—Games.)

You can keep naughty names from appearing on the Hall of Fame by not allowing students to create their own data files. This is accomplished by adding all students to the class list and then filling up the disk with dummy data files to disable the "NEW STUDENT" routine. The disk holds 50 data files. Once all 50 files have been created, the "NEW STUDENT" routine will not allow another name to be entered.

How Do I Change a Student's Name, Grade, or Group?

If a mistake has been made in entering a student's name or grade level, the mistake can be fixed without affecting other data in the file. This mistake can be corrected by using the appropriate change routine. Do not fix mistakes by deleting and then re-creating the student file, as this practice will also delete previously collected data.

Floppy Disk Version: Go to the Tracking Menu. Select the "Change Student's Name/Grade" option. Use the arrow keys to select the student whose name or grade should be changed. Enter a new first name, last name, and grade level. Change another student's name or press the escape key to quit.

M. D. Network Version: Go to the Teacher's Menu on the Teacher Station computer. Select the "Change Files" option. Enter "G" to change a student's group. Enter "N" to change a student's name. Follow the directions displayed in the explanation box at the top of the screen.

How Many Problems Are Presented in Each Session?

A session consists of four screens of six problems each, or 24 problems. At the end of each session, students are allowed to play the game with the game pieces they have earned. At the end of each game, the students are asked if they would like to continue to play the game. If they respond "Yes," they are presented with another session of facts from the same skill. If they respond "No," the program returns to the Main Menu, where the student can choose a new skill or can relinquish the computer to another student. Students are encouraged to complete multiple sessions (three minimum) with the same skill before changing skills or trading places with another student.

How Can Students Most Quickly Advance To the Next Level?

All students are initially assigned to level one (Beginner) for each skill. If a student performs well on his first attempt, he/she is given an early advancement to level two (intermediate). Otherwise, in order to advance to the next level, students must master most of the problems in their current level. Students diligently working on a single skill will quickly (six to nine sessions) advance to their proper level.

NOTE: In order to quickly advance to the proper level it is important that students do not jump between skills (i.e., play the addition game once, then the subtraction game once, then the multiplication game, then back to the addition game, etc.), or alternate playing games with one or more students. Jumping around retards the advancement process because the computer will flush the memory of all intermediate results (i.e., in-progress facts which have been mastered once or twice) each time a new student plays a game, or when a student works on more than two different skills in succession. In other words, a student who works straight through on addition (or intersperses work on addition with no more than one other skill) will be able to carry the facts mastered once or twice into the next session. On the other hand, the student who works on more than two skills, or rotates playing with another student, must begin all unmastered facts at the first try. Moreover, the chances that the student will be presented with any problem three times in a single session is slim. Therefore, encourage students to play multiple sessions (three minimum) on a single skill before switching skills or relinquishing their position at the computer. Hint: Some teachers set all the skills to the same game (i.e., game of the week) to discourage students from hopping between games.

What Is the Mastery Development School Tracking Network System?

The Mastery Development School Tracking Network System was designed especially for elementary schools and middle schools. It is lightning fast, easy to use, and contains all Mastery Development software (i.e., *Mastery Math, Mastery Arithmetic, Math Facts Tracker, Keyboarding Klass, The Writing Room, Geo-Race U. S. A, Geo-Race Europe, Geo-Quiz, Spellbinders,* and additional products each year), most of the MECC software, and hundreds of other titles. It eliminates the need to distribute and boot individual floppy disks. Moreover, all of the Mastery Development products take full advantage of the increased storage capacity of the network system to provide tracking and reporting features not possible on the floppy disk version. If you would like more information on this network system, call or write for the Network Brochure and the Network Additional Information Booklet.

REFERENCES

Citations

Armstrong, T. (1983) Chasing away the times tables blues. *Academic Therapy*, **19:2**, 147–154.

Ashcraft, M. H., & Battaglia, J. (1978). Cognitive arithmetic: Evidence for retrieval and decision processes in mental addition. *Journal of Experimental Psychology: Human Learning & Memory*, **9**, 185–196.

Ashlock, R. B., & Washbon, C. A. (1978). Games: practice activities for the basic facts. *Developing Computational Skills 1978 Yearbook*. pg 39–49. National Council of Teachers of Mathematics, Reston, Virginia, 1978.

Baroody, A. J. (1983). The development of procedural knowledge: An alternative explanation for chronometric trends of mental arithmetic. *Developmental Review*, **3**, 225–230.

Baroody, A. J. (1985). Mastery of basic number combinations: Internalization of relationships of facts? *Journal for Research in Mathematics Education*, **16**, 83–98.

Baroody, A. J. (1987). *Children's Mathematical Thinking*. New York, NY: Teachers College Press

Baroody, A. J. (1988). Mental-Addition development of children classified as mentally handicapped. *Educational Studies in Mathematics*, **19**, 369–387.

Baston W. H., & Combellick). E. (1925). Relative difficulty of number combinations in addition and multiplication. *The Journal of Educational Psychology* , **16**, 467–481.

Brownell, W. A. (1944). *Arithmetic in Grades I and II*, Duke University Research Studies in Education #6–8, pg 116–127. Durham, Duke University Press, 1941.

Brownell, W. A., & Carper D. V. (1944). *Learning the Multiplication Combinations*. Duke University Research Studies in Education #5, Durham, Duke University Press, 1941.

Brownell, W. A., & Chazal, C. B. (1935). The effects of premature drill in third-grade arithmetic. *Journal of Educational Research*. **29**, 17–28.

Clapp, F. L. (1924). *The number combinations their relative difficulty and the frequency of their appearance in text-books*. Bureau of Educational Research, Bulletin No. 1–2, Madison, Wis.: University of Wisconsin, 1924.

Dumas, E., & Schminke, C. W. (1977). *Math activities for child involvement*. Boston: Allyn and Bacon.

Effros, E. G. (1989). Perhaps nobody will count. *Education week*. Apr 12, 1989, 36.

Fuson, K. C., & Willis, G. B. (1988). Subtracting by counting up: More evidence. *Journal for Research in Mathematics Education*. **19**, No. 5, 402–420.

Groen, G. J. & Parkman, J. M. (1972). A chronometric analysis of simple addition. *Psychological Review*, **79**, 329–343.

Hamann, M. S., & Ashcraft, M. H. (1985). Simple and complex mental addition across development. *Journal of Experimental Child Psychology*, **40**, 49–72.

Klum, G. (1985). Strategies and activities—learning to add and subtract. ERIC document ED226945.

Miller, K., Perlmutter, M., & Keating, D. (1984) Cognitive arithmetic: Comparison of operations. *Journal of Experimental Psychology: Learning, Memory, and Cognition*. **10**, 46–60.

Parkman, J. M. & Groen, G. J. (1971). Temporal aspects of simple addition and comparison. *Journal of Experimental Psychology*, **89**, 335–342.

Rathmall, E. C. (1978). Using thinking strategies to teach the basic facts. *Developing Computational Skills 1978 Yearbook*. pg 13–38. National Council of Teachers of Mathematics, Reston, Virginia, 1978.

Resnick, L. B., & Ford, W. W. (1981). *The psychology of mathematics for instruction*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Salisbury, D. F. (1990). Cognitive psychology and its implications for designing drill and practice programs for computers. *Journal of Computer-Based Instruction*. **17**(1), 23–30.

Schneider, W., & Shiffrin, R. M. (1977). Controlled and automatic human information processing: I. Detection, search, and attention. *Psychological Review*, **84**, 1–66.

Schneider, W., & Shiffrin, R. M. (1977). Controlled and automatic human information processing: II. Perceptual learning, automatic attending, and a general theory. *Psychological Review*, **84**, 127–190.

Siegler, R. S., & Shrager, J. (1983). Strategy choices in addition: How do children know what to do? In C. Sophian (Ed.), *Origins of cognitive skills: The 18th Annual Carnegie Symposium*. Hillsdale, NJ: Earlbaum.

Steinberg, R. M. (1985) Instruction on derived facts strategies in addition and subtraction. *Journal for Research in Mathematics Education*, **16**, 337–355.

Steinberg, R. M., Baroody, A. J., & Fuson, K. C. (1984). From counting to recall of number facts. In J. Moser (Ed.), *Proceedings of the Sixth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (253–262). Madison: University of Wisconsin, Wisconsin Center for Education Research.

Svenson, O., & Sjoberg, K. (1982). Solving simple subtractions during the first three school years. *Journal of Experimental education*, **50**, 91–100.

Svenson, O., & Sjoberg, K. (1983). Evolution of cognitive processes for solving simple additions during the first three school years. *Scandanavian Journal of Psychology*, **24**, 1–8.

Swenson, E. J. (1949). Organization and generalization as factors in learning, transfer and retroactive inhibition. *Learning Theory in School Situations*. University of Minnesota Studies in Education. no 2. Minneapolis: University of Minnesota Press.

Taut, K., Hartley, J. R., & Anderson, R. C. (1973) Feedback procedures in computer-assisted arithmetic instruction. *British Journal of Educational Psychology*, **43**, 161–171.

Thiele, C. L. (1938). The contribution of generalization to the learning of the addition facts. *Contributions to Education no. 763*. New York Bureau of Publications, Teacher's College, Columbia University.

Thornton, C. A. (1989). "Look Ahead" Activities spark success in addition and subtraction number-fact learning. *Arithmetic teacher*. April, 1989. pg 8–11.

Thornton, C. A., & Toohey, M. A. (1984) *Matter of Facts: Addition*. Palo Alto, Calif. Creative Publications, 1984

Wheeler L. R. (1939) A comparative study of the difficulty of the 100 addition combinations. *The Journal of Genetic Psychology*, **54**, 295–312.

Other Helpful References

Cook, C. J. & Dossey, J. A. (1982). Basic fact thinking strategies for multiplication revisited. *Journal for Research in Mathematics Education*, **13**, 163–171.

Hebbeler, K. M. (1978). The development of addition problem solving in young children. (Doctoral dissertation, Cornell University, 1978). *Dissertation Abstracts International*, **xxx**.

Jerman, M. (1970). Some strategies for solving simple multiplication combinations. *Journal for Research in Mathematics Education*. **1**, 95–128.

Randall, I. C. (1986). Fact strategies–Bridging the way to better math performance. *Learning*, Jan. 1986, pg. 54–60.

Steinberg, R. (1984). A teaching experiment of the learning of addition and subtraction facts. (Doctoral dissertation, University of Wisconsin—Madison, 1983). *Dissertation Abstracts International*, **44**, 3313A.

I am always interested in learning of new research about how children learn math facts. If you have conducted any such research, or have the references of good research studies, please share them with me. In return, I'll be pleased to share any new information I have with you.

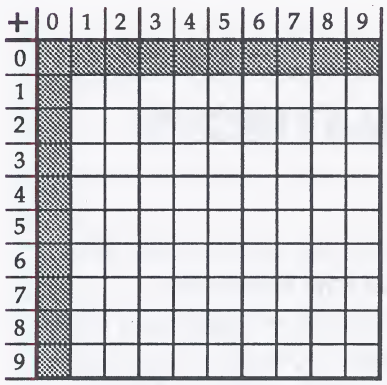
- J. Scott Harris, Mastery Development • P. O. Box 3126 • Redmond, Washington • 98073

APPENDIX

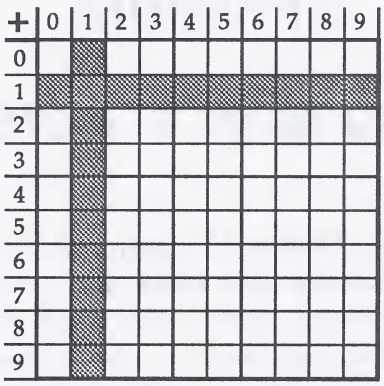
STRATEGY AND M. F. T. LEVEL MATRICES

This Appendix contains Strategy Usage, Dominant Strategy, and M. F. T. Level matrices for each of the four operations. The Strategy Usage matrices show which facts can be solved with each of the relational strategies. The Dominant Strategy matrix shows which of the relational strategies are most likely to be employed by children as they progress from counting to automatization/recall. Counting strategies and groups of easily memorized facts also appear in the Dominant Strategy matrix, because children progress from counting directly to memorization for many of the math facts. The M. F. T. Level matrix shows which facts are grouped in each of the four Math Fact Tracker levels– Beginner, Intermediate, Advanced, and Expert.

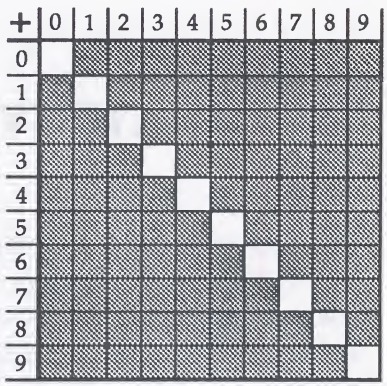
Addition—Strategy Usage



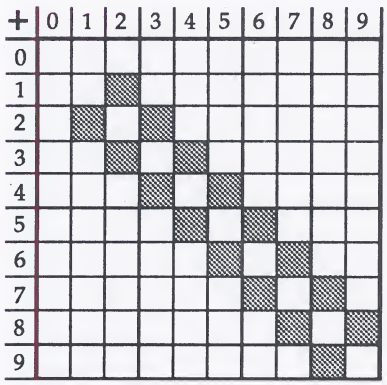
1. Zero Rule for Addition



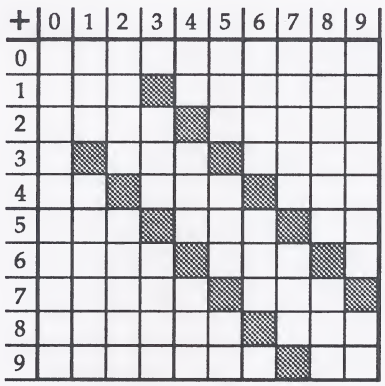
2. One's Rule for Addition



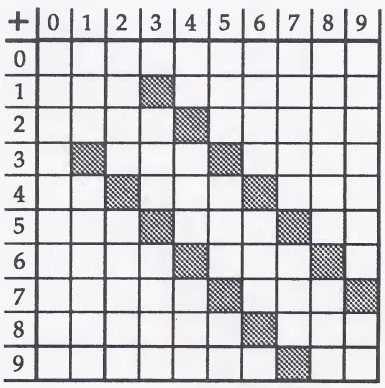
3. Commutative Property



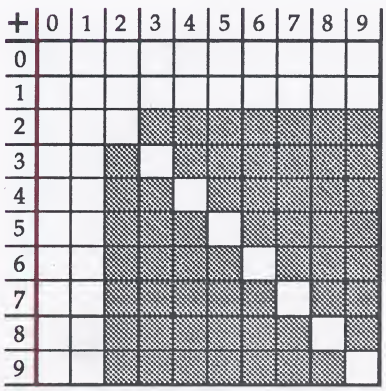
4. Doubles +1 or -1



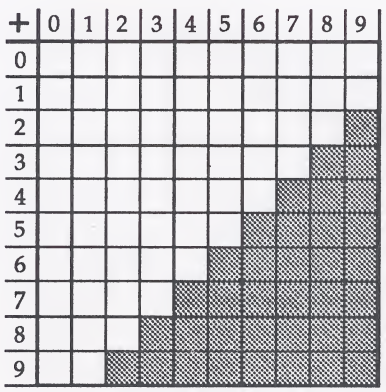
5. Doubles +2 or -2



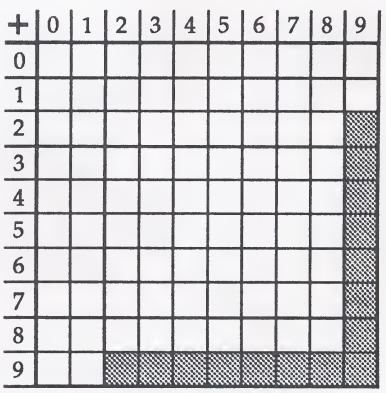
6. Sharing



7. Known Fact +1 or -1



8. Over-Ten



9. Nines Trick for Addition

Addition—Dominant Strategies

Addend 1

Addend 2	0	1	2	3	4	5	6	7	8	9
0	1	1	1	1	1	1	1	1	1	1
1	1	*	2	2	2	2	2	2	2	2
2	1	2	*	4 C	5 6 C	C	C	C	C	C 9
3	1	2	4 C	*	4 C	5 6 C	C	C	C 8	C 9
4	1	2	5 6 C	4 C	*	4	5 6	7 8	7 8	9
5	1	2	C	5 6 C	4	*	4 8	5 6 8	7 8	9
6	1	2	C	C	5 6	4 8	*	4 8	5 6 8	9
7	1	2	C	C	7 8	5 6 8	4 8	*	4 8	9 5 6
8	1	2	C	C 8	7 8	7 8	5 6 8	4 8	*	9 4
9	1	2	C 9	9	9	9	9	9 5 6	9 4	* 9

COUNTING STRATEGIES

A. Count-All
B. Count-on-from-First
C. Count-on-from-Largest

RELATIONAL STRATEGIES

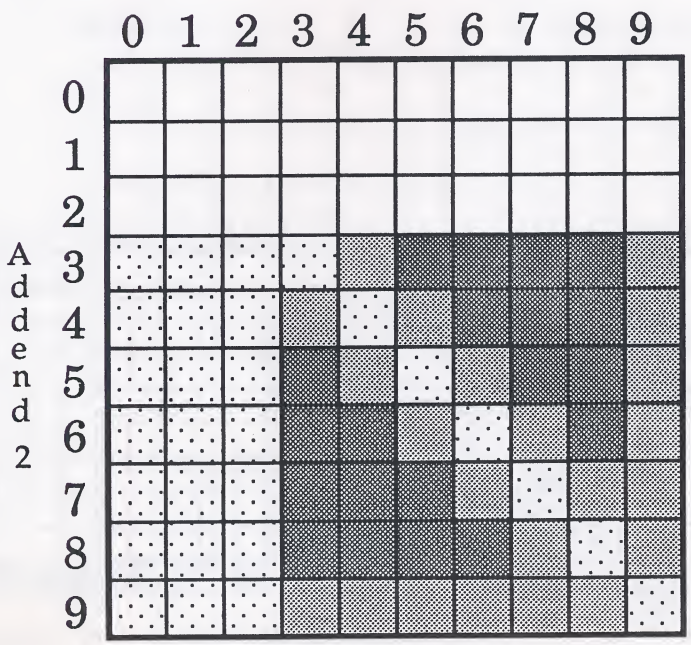
1. Zero Rule for Addition
2. Ones Rule for Addition
3. Commutative Property
4. Doubles +1 or -1
5. Doubles +2 or -2
6. Sharing
7. Known Fact +1 or -1
8. Over-Ten
9. Nines Trick for Addition

RECALL

* Doubles

Addition—M. F. T. Levels

Addend 1



9 Addend 1
+3 Addend 2
12 Sum

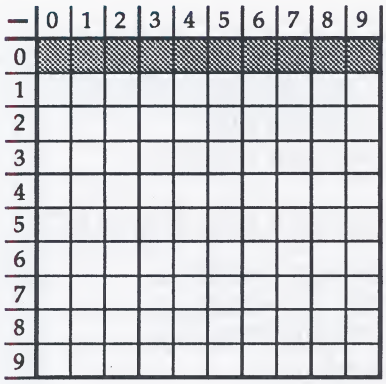
= Level 1 (Beginner)

= Level 2 (Intermediate)

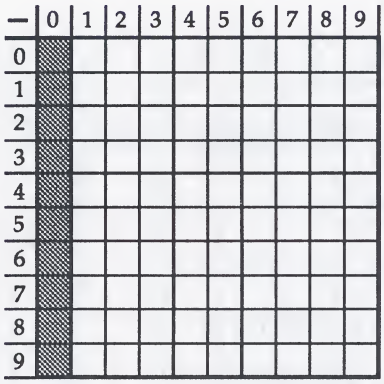
= Level 3 (Advanced)

= Level 4 (Expert)

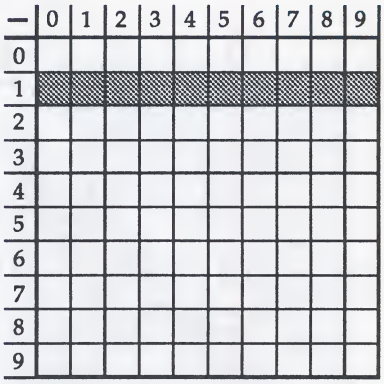
Subtraction—Strategy Usage



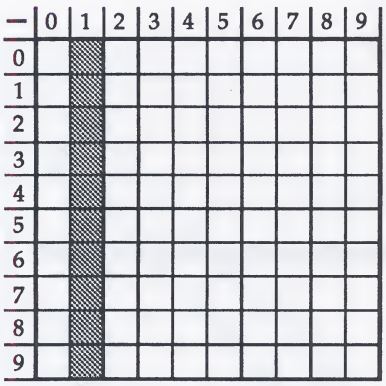
1. Zero Rule for Subtraction



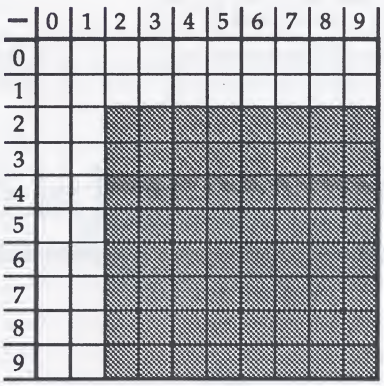
2. Doubles Rule for Subtraction



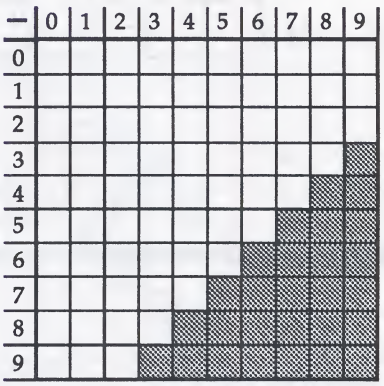
3. Ones Rule for Subtraction



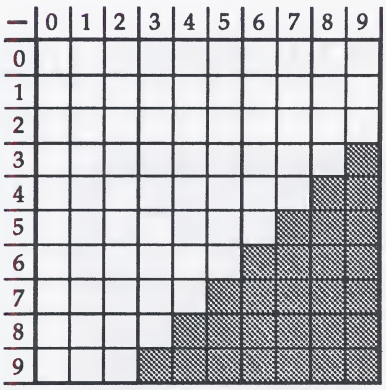
4. Neighbors Rule



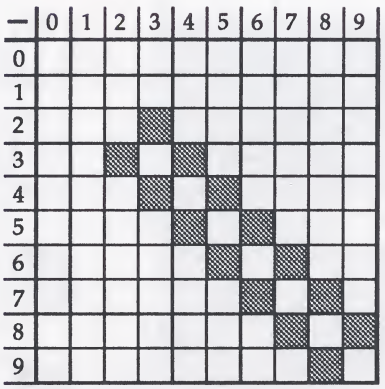
5. Fact Families



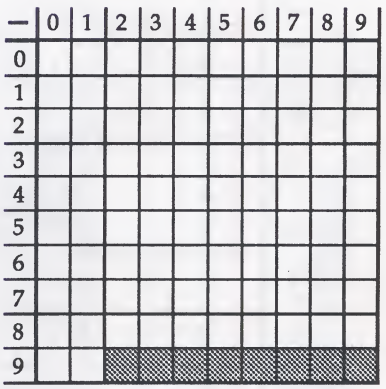
6. Up-Over-Ten



7. Down-Past-Ten



8. Compensation



9. Nines Trick for Subtraction

Subtraction—Dominant Strategies

Subtrahend \ Difference	0	1	2	3	4	5	6	7	8	9
0	1 2	1	1	1	1	1	1	1	1	1
1	2	3 4	3	3	3	3	3	3	3	3
2	2	4	*	C B	B	B	B	B	B	B
3	2	4	C	*	B 8	B	B	B	B	6 7 B
4	2	4	C	C 8	*	8	5	5	6 7	6 7
5	2	4	C	C	8	*	8	6 7	6 7	6 7
6	2	4	C	C	5	8	*	6 7 8	6 7	6 7
7	2	4	C	C	5	6 7	6 7 8	*	6 7 8	6 7
8	2	4	C	C	6 7	6 7	6 7	6 7 8	*	6 7
9	2	4	C	9 C	9	9	9	9	9	* 9

COUNTING STRATEGIES

A. Take-Away
B. Count-Down
C. Count-Up

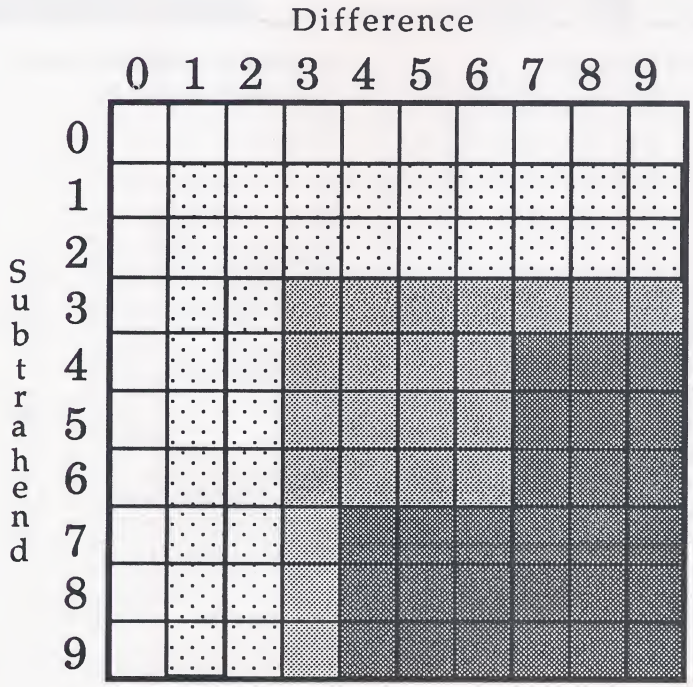
RELATIONAL STRATEGIES

1. Zero Rule for Subtraction
2. Doubles Rule for Subtraction
3. Ones Rule for Subtraction
4. Neighbors Rule
5. Fact Families
6. Up-Over-Ten
7. Down-Past-Ten
8. Compensation
9. Nines Trick for Subtraction
0. Scott's Method

RECALL

* Reverse Doubles

Subtraction—M. F. T. Levels



13 Minuend
-8 Subtrahend
5 Difference

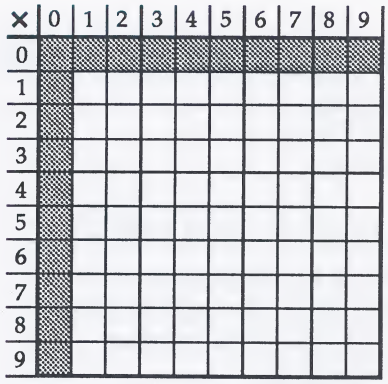
= Level 1 (Beginner)

= Level 2 (Intermediate)

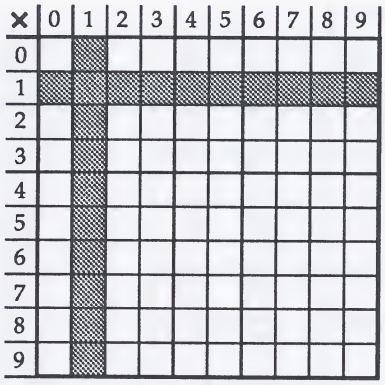
= Level 3 (Advanced)

= Level 4 (Expert)

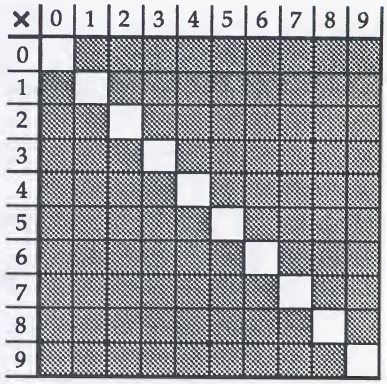
Multiplication—Strategy Usage



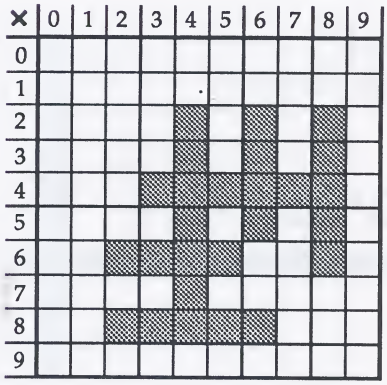
1. Zero Rule for Multiplication



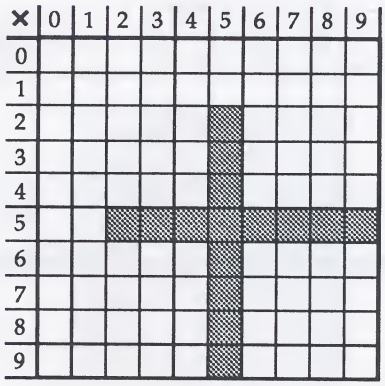
2. Ones Rule for Multiplication



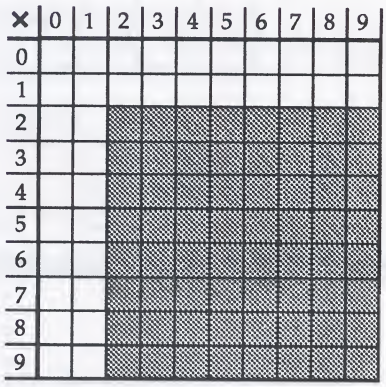
3. Commutative Property



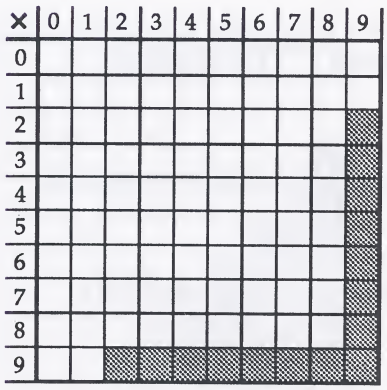
4. Recall Half then Double



5. Halving



6. Add or Subtract from a Known Product



7. Nines Trick for Multiplication

Multiplication—Dominant Strategies

Multiplicand

Multiplier	0	1	2	3	4	5	6	7	8	9
0	1	1	1	1	1	1	1	1	1	1
1	1	2	2	2	2	2	2	2	2	2
2	1	2	*	*	*	* #	* 4	*	* 4	* 7
3	1	2	*	r	4	# 5	4	A	4	7
4	1	2	*	4	4	# 4 5	4	4	4	7
5	1	2	* #	# 5	# 4 5	# 5	# 5	# 5	# 5	7 # 5
6	1	2	* 4	4	4	# 5	6	6	4 6	7
7	1	2	*	A	4	# 5	6	6	6	7
8	1	2	* 4	4	4	# 5	4 6	6	6	7
9	1	2	* 7	7	7	7 # 5	7	7	7	7

COUNTING STRATEGIES

A. Repeated-Addition
B. Array-Counting
C. Skip-Counting

RELATIONAL STRATEGIES

1. Zero Rule for Multiplication
2. Ones Rule for Multiplication
3. Commutative Property
4. Recall Half then Double
5. Halving
6. Add or Subtract from a Known Product
7. Nines Trick for Multiplication

RECALL

* Twos Facts
\# Fives Facts

Multiplication—M. F. T. Levels

Multiplicand

Multiplier	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3										
4										
5										
6										
7										
8										
9										

7 Multiplicand
x3 Multiplier
21 Product

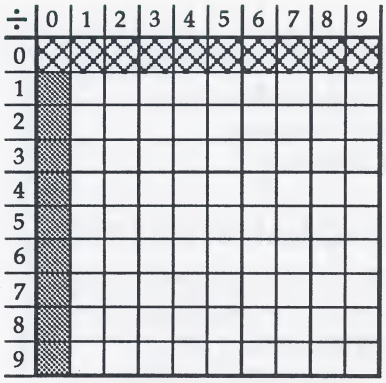
= Level 1 (Beginner)

= Level 2 (Intermediate)

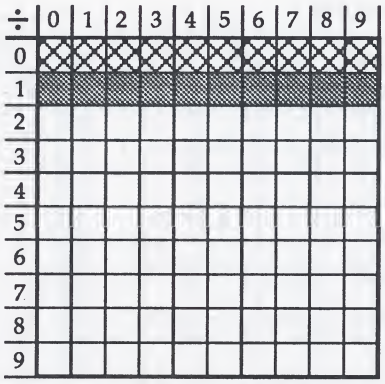
= Level 3 (Advanced)

= Level 4 (Expert)

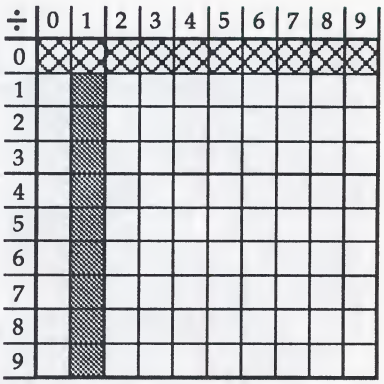
Division—Strategy Usage



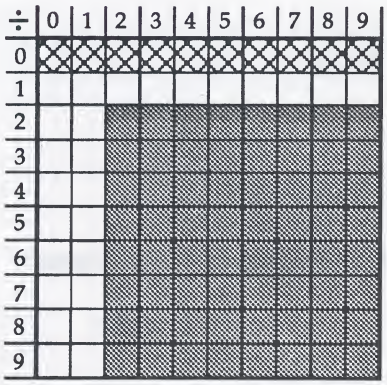
1. Zero Rule for Division



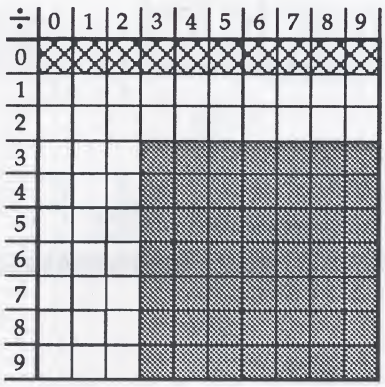
2. Ones Rule for Division



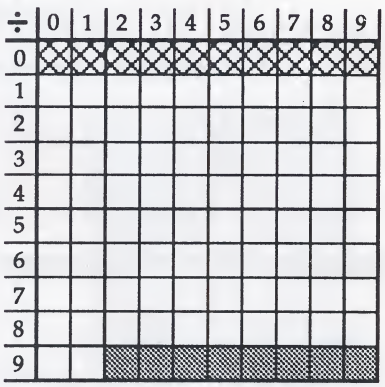
3. Doubles Rule for Division



4. Fact Families



5. Add or Subtract from a Known Product



6. Nines Trick for Division

Division—Dominant Strategies

Divisor \ Quotient	0	1	2	3	4	5	6	7	8	9
0										
1	1	2 3	2	2	2	2	2	2	2	2
2	1	3 *	* 4	* 4	* 4	* 4	* 4	* 4	* 4	* 4
3	1	3	4	4	4	4	4	4	4	4
4	1	3	4	4	4	4	4	4	4	4
5	1	3	# 4	# 4	# 4	# 4	# 4	# 4	# 4	# 4
6	1	3	4	4	4	4	4 5	4 5	4 5	4 5
7	1	3	4	4	4	4	4 5	4 5	4 5	4 5
8	1	3	4	4	4	4	4 5	4 5	4 5	4 5
9	1	3	6	6	6	6	6	6	6	6

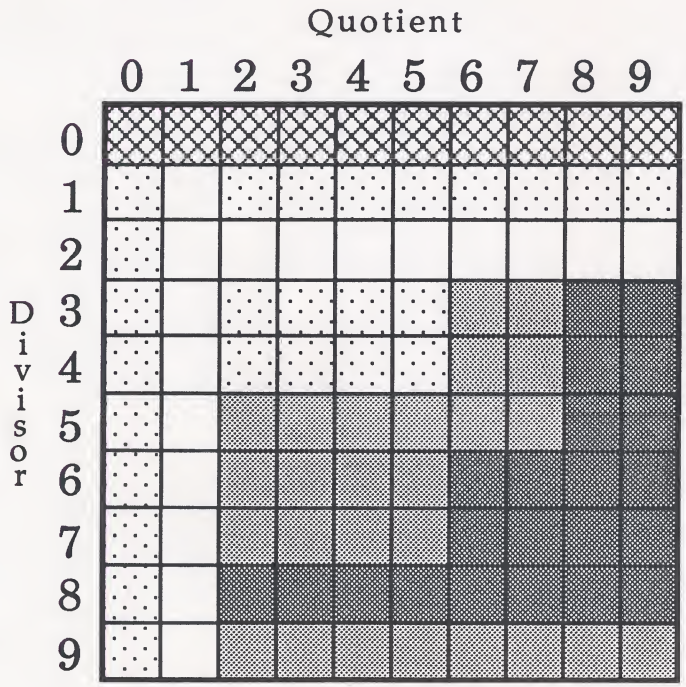
RELATIONAL STRATEGIES

1. Zero Rule for Division
2. Ones Rule for Division
3. Doubles Rule for Division
4. Fact Families
5. Add or Subtract from a Known Product
6. Nines Trick for Division

RECALL

* Twos Facts
\# Fives Facts

Division—M. F. T. Levels



$$\begin{array}{l} \text{Divisor } 6\overline{)24} \\ \text{Quotient } 4 \\ \text{Dividend} \end{array}$$

= Undefined Operation

= Level 1 (Beginner)

= Level 2 (Intermediate)

= Level 3 (Advanced)

= Level 4 (Expert)